PROBABILISTIC CFD AND EVACUATION SIMULATION FOR FIRE LIFE SAFETY ASSESSMENT

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ABSTRACT

The process of designing for life safety is usually performed using parameters which are known to be subjected to high uncertainties. Hence, the choice of the appropriate deterministic parameters is usually very conservative to envelope those uncertainties. Until recently, the probabilistic assessment of life safety using CFD and microscopic evacuation models only seemed to be theoretically possible as a Monte Carlo simulation of CFD-based fire design is unfeasible due to the high computational cost. In this paper we present a fast and efficient adaptive response surface approach to perform probabilistic life safety analysis using state-of-the-art fire engineering tools, namely the CFD fire simulation code FDS and the built-in FDS+evac microscopic evacuation software, to assess the reliability of life safety during a hostile fire. This approach also allows for sensitivity analysis to identify the critical input parameters. As installed protection barriers (i.e. sprinklers etc.) are still prone to possible failure, a subsequent system analysis can yield information about the total fire protection system reliability considering the possible malfunction of the barriers. Additionally, the identification of the most critical scenarios is possible. The methodology will be exemplary applied to a small size assembly building for multiple scenarios and barriers, and the results will be discussed in detail. Probabilistic assessment of the life safety using state-of-the-art engineering tools delivers valuable information about the behavior of a fire protection system, influence of the various possible barriers and it allows for a quantitative comparison of multiple strategies in order to identify the most cost-effective solution.

METHODOLOGY

As the approach described above is usually applied using deterministic values of an uncertain variable, engineers tend to estimate values on the safe side and thus might end up with overly safe and expensive solutions. The aim of this paper is to compute the reliability of a safe evacuation using a CFD model and a simple evacuation calculation. As a Monte Carlo simulation of a CFD model is not possible due to the high computational costs, a response surface method based on moving least squares was chosen to minimize the number of necessary solver evaluations. To further decrease the necessary number of numerical evaluations a preceding sensitivity analysis yields information about the variance of the results and the relevance of the input parameters and hence helps to identify a surrogate model of optimal prognosis. The preliminary scan of the random space for the sensitivity analysis can also be used to obtain information about the approximate location of the design point so that further support points can be concentrated in this relevant area. Using this information for the subsequent reliability analysis leads to a faster convergence.

Sensitivity Analysis

In order to perform a sensitivity analysis of the variance of the input vs. the output variables, a preliminary scan of the random hyperspace has to be performed using all random variables considered. The points of the scan can either be chosen randomly or systematically, using common design of experiment (DoE) plans such as the Central Composite design (Forrester, 2008). The input data can then be linked to the corresponding output quantity and simple, global sensitivity analysis can be performed:

Correlation Analysis

A first analysis is performed to identify the significant contributors to explain the output variability. This is done by linear and rank correlation analysis and a subsequent test of the t_{ij} -value against a chosen significance level $t_{\alpha/2}$ of the Student's distribution. If $|t_{ij}| > t_{\alpha/2}$ the null hypothesis that there is no significant correlation is rejected in favor of the alternative hypothesis that a significant correlation occurs (Mendenhall, 1996). This test can mainly be used to identify the significant variables and the occurrence of a linear correlation or a rank correlation if the data is rank transformed (Schwieger, 2005).

Stepwise Regression Analysis

In order to find an optimal surrogate model, also interaction and correlation effects between the input parameters have to be considered in order to find the global contribution to the prediction accuracy of each variable. This is done by considering the adjusted coefficient of determination (Mendenhall, 1996)

$$R_{adj}^{2} = 1 - \frac{n-1}{n-(k+1)} (1-R^{2}), R_{adj}^{2} \in [0,1]$$
(1)

of a simple linear and/or rank regression model where *n* is the number of samples and *k* the number of parameters so that (k+1) denotes the degrees of freedom. R^2 is the coefficient of determination usually applied to larger datasets. R^2_{adj} penalizes for the number of variables when considering smaller datasets (such as a DoE).

The stepwise regression approach by Draper (1998) uses this methodology by subsequently adding variables to the regression model and performing a global *F*-test as described in Mendenhall, 1996, looking at the change in the coefficient of determination (ΔR^2_{adi}) (Most, 2008).

Applying these methods usually yields that a few variables are unimportant as they have none or very little effect on the variance of the output. Hence, for further analysis they can be chosen as a deterministic value by using i.e. the mean value. A variable and thus dimensional reduction of the reliability problem can significantly reduce the number of necessary solver evaluations in the design of experiment scheme. It should be noted that all variables to be removed should also be checked qualitatively before removal as the methods described above only provide a purely mathematical approach.

Reliability Analysis

Surrogate Model

Reliability analysis will be carried out using a surrogate model which is deduced from the support point values of the design of experiment evaluations. Common response surface methods such as by Bucher (1990) utilize least square regression to fit a polynomial model in the form

$$\hat{f}(\mathbf{x}, \hat{\boldsymbol{\beta}}) = \mathbf{y} = \mathbf{H}\hat{\boldsymbol{\beta}} + \boldsymbol{\varepsilon}$$
 (2)

to the support points and subsequently uses the analytical equation found in first order reliability method (FORM) to evaluate the failure probabilities. Herein, **H** is a matrix of *n* functions and $\hat{\mathbf{\beta}}$ is a vector

of *n* free coefficients to be fitted by minimizing the error component ε (Draper, 1998). $\hat{\beta}$ can be found (proof omitted) by

$$\boldsymbol{\beta} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{y} \,. \tag{3}$$

The downside of this approach is that only global trends can be considered and information at the computationally expensive support points is only approximated. A very high order polynomial meets the interpolation conditions but tends to have an over-fitting effect between the supports (Forrester, 2008).

The approach followed herein is a moving least square (MLS) approach which is based on an enhancement of the above concept of least squares by Lancaster & Salkauskas (1981) by incorporating location information to increase the accuracy of the approximation. The approach utilizes a weighting of the Eucledian distance of each support point input vector \mathbf{x}_{mi} to the input parameters \mathbf{x} of the evaluation point so that

$$w_i(\mathbf{x}, \mathbf{x}_{mi}) = w_i(||\mathbf{x} - \mathbf{x}_{mi}||), \qquad (4)$$

All weights are then compiled into a location dependent weighting matrix W(x) which can be introduced into eq. 3 so that

$$\hat{\boldsymbol{\beta}}(\mathbf{x}) = [\mathbf{H}^T \mathbf{W}(\mathbf{x})\mathbf{H}]^{-1}\mathbf{H}^T \mathbf{W}(\mathbf{x})\mathbf{y}.$$
 (5)

Unfortunately, this leads to a location dependency of the coefficients $\hat{\beta}$ so that no closed-form global equation can be found. A MLS formulation has to be found for every evaluation point.

The weighting function described in eq. 4 is a radial function which must be greater than zero, symmetric around the support point, and monotonically decreasing. Usually, cubic polynomials (Kunle, 2001) or Gaussian curves (Most, 2005) are utilized, but both do not fulfill the Kronecker-Delta properties

$$w_i(\|\mathbf{X} - \mathbf{X}_{mi}\|) \approx \delta_{ij} \tag{6}$$

required for interpolation (Bronstein, 2008).

An approach outlined in (Bucher,2009,Most, 2005) uses a nearly interpolating weighting function by introducing

$$w_i(||\mathbf{x} - \mathbf{x}_{mi}||) = \frac{\tilde{w}_r(||\mathbf{x} - \mathbf{x}_{mi}||)}{\sum_{j=1}^m \tilde{w}_r(||\mathbf{x} - \mathbf{x}_{mi}||)}$$
(7)

with

$$\tilde{w}_{r}(\|\mathbf{x}-\mathbf{x}_{mi}\|) = (\|\mathbf{x}-\mathbf{x}_{mi}\|^{2} + \varepsilon)^{-2}, \varepsilon \ll 1$$
(8)

where ε is a regularization parameter to stabilize the problem numerically. In order to provide near-accurate solutions ε has to be chosen as small as possible, but within machine precision. An ε in the range of 10E-5 is usually sufficient (Most, 2005).

Adaptive Importance Sampling

As described above, the fitted coefficients of an MLS approach are now location dependent and thus no closed-form global expression is available. Hence, Proppe (2008) recommends the use of adaptive importance sampling (AIS) instead of FORM. The basic idea is to reduce the variance $\hat{\sigma}_{p_e}$ by introducing a weighting function h_x into a Monte Carlo Simulation so that the sampling points are concentrated in the failure domain Ω_f . h_x is adapted in subsequent iteration steps by only considering the points that fell into the failure domain. Further adaptations of h_x lead to a smaller variance and hence better results. AIS is described in detail in (Bucher, 1988). Ibidem, empirical sensitivity factors are derived by looking at the total shift of the mean vector in h_x compared to the joint density function $f_{\rm x}$. The sensitivity of each variable can be regarded as the contribution to the total shift.

Adaptivity

As the reliability evaluation is performed using the surrogate model and not the real (CFD, evacuation) model, some inaccuracies between approximation and model can occur, especially for highly non-linear problems in the regions without close-by support points. Hence, in a next step the support points will be updated around the mean vector from that last adaptation of h_x . These support points in the relevant region along with a smaller variance, also provided by h_x , yield a very good approximation of the limit state hyper-surface so that usually only a few

additional support points are necessary. The advantage of this approach is that all previous evaluations can be re-used in the next step surrogate design so that no "expensive" information is discarded. The local approximation quality in the failure area stays high due to the nearly-fulfilled interpolation conditions.

The difference between the iterations can be used as a convergence criteria. If the change in $\hat{p}_{f,i}$ is less than 2.5% compared to $\hat{p}_{f,i-1}$ of the previous iteration step, the algorithm is terminated.

System Analysis using Event Trees

Event trees are used herein to model the various possible scenarios that potentially lead to failure. Event trees allow for an easy scenario generation and include the chronological order of events to occur. A very simplified example is shown in fig. 1. Usually, each branch of an event tree denotes a cut set of the system. This can be visualized by transforming the event tree to a fault tree.

Looking at fig. 1, the horizontal direction denotes roughly multiple parallel systems (each branch). Possible correlation effects or interaction effects between the barriers, respectively, are modeled within the scenarios and thus are approximately accounted for. In the vertical direction, an uncorrelated series system of the multiple scenarios can be assumed, as the possible function or malfunction of the barriers is purely stochastic (the "way" or "route" through the event tree is a random process) with the according probabilities.

Adding the probabilities of each branch yields a conservative result for the system reliability, as this constitutes the first order upper bound for series systems. For small systems, this approximation is usually sufficient as it is very close to the exact solution.



Figure 1: Modeling of Fire Protection Barriers in an Event Tree

APPLICATION TO LIFE SAFETY

The main focus of the application example is to evaluate to reliability of a save evacuation from an assembly room. This is usually shown if the required safe egress time (RSET) is smaller than the available safe egress time (ASET) so the limit state of the reliability problem can be simply derived as

$$g(\mathbf{x}) = t_{\text{ASET}} - t_{\text{RSET}} \tag{9}$$

with a failure domain $\Omega_f \equiv g(\mathbf{x}) \leq 0$. The times stem from an evacuation model using the microscopic model FDS+evac and a CFD fire simulation using FDS in the latest version, respectively. The simulation runs will be performed with various random variables shown in the following and the results of those simulations will subsequently be evaluated for the needed time spans.

A medium size assembly building

To provide a representative and no to complex example, a medium size assembly building was chosen for further evaluation. This seems reasonable as a large number of this type of buildings exist throughout nearly all countries of the world. While the fire protection in large shopping centers is usually maintained to a certain degree it is often disregarded at smaller to medium assembly buildings. The outline of the chosen nightclub-type assembly building is shown in fig. 2.

The building has three exits. The main exit at the west side is adjacent to a coatroom and is separated from the main bar/dance area. This layout generally yields into slower evacuation due to congestion. Among others, this was the reason for the disastrous outcome of the Station Fire, as many victims were found near the main exits (Bryner, 2007). The other too exits are emergency exits located north and east and are considered clearly marked but unknown to many of the occupants which can be modeled as "familiarity" in FDS+evac.

Table 1: Stochastic models for the uncertain variables.

Also many obstructions can be found within the building: near the bar tables and bar stools scattered and the dance floor is separated from the rest of the nightclub with guardrails. For the evacuation scenarios the highest occupant density was assumed on the dance floor, followed by the area around the bar.



Figure 2: Floor plan of the nightclub-type assembly building.

Stochastic Modeling

Human behavior is not only highly subjected to uncertainties but also dependent on various parameters which, again, are dependent on the evacuation model used which herein in the microscopic FDS+evac model. This model uses the equation of motion as basis for the movement of individual agents. Interactions are modeled using socalled "social forces". Further details can be found in Helbing (1999).

For the approach herein, the number of occupants, warning and pre-movement times, anticipated velocity of the agents, and the shoulder width were identified as the parameters with the highest uncertainties and thus were described with stochastic model. These can be found in tab. 1.

For the fire simulation, the CFD software FDS was utilized. Herein, the maximum heat release rate (HRR), the fire growth rate (herein expressed as time

Parameter	Unit	Dist. type	Mean	StdDev.
HRR	kW/m²	Normal	500	100
t _g	s	Gumbel (min)	250	50
Усо	g/g	Normal	0.090	0.030
y _{HCN} (as "other" species")	g/g	Normal	0.006	0.002
Y_Soot	g/g	Normal	0.120	0.040
# Occupants	occ.	Gumbel (max)		
Warning time	S	Normal	60	15
Pre-move. time	s	Gumbel (max)	90/180	25/45
Velocity	m/s	Normal	1.25	0.3
Shoulder width	m	Normal	0.51	0.07

in seconds until 1 MW is reached), and the yields for carbon monoxide (CO), hydrogen cyanide (HCN), and soot were taken as uncertain parameters as shown in tab. 1.

Unfortunately, stochastic models rarely exist in the relevant literature and even deterministic values can be difficult to find. Tab. 1 shows the stochastic models used herein which are usually based on deterministic sources and an educated guess of the occurring variance, where it is usually assumed that the variables have a standard deviation of 10-20% about the mean.

Threshold models for ASET and RSET

When using performance-based methods for life safety design, thresholds have to be defined in order to establish a tenability limit in the fire simulation (ASET). These thresholds have been widely discussed in the literature such as Purser (2002). Herein, we will use an optical density threshold of 0.1/m and will also look at the combined cumulative exposure to toxic fire effluents, heat, and irritant gases (FED of 1.0 including a conservatively assumed lump-sum for irritant gases of 0.3). The definitions of these criteria can be found in detail in the literature cited above.

A subject to discussion is usually the location within the fire simulation where the threshold criteria is recorded. Usually this is done by looking at slice files in a height of 2.0m or 2.5m to incorporate some safety. If the tenability limit is qualitatively reached in the greater part of the compartment the ASET is set (Mahlmann, 2009). Herein, we use a less arbitrary strategy which also stabilizes the variance of the results. Volumes of multiple CFD cells are chosen spanning about 5m² and a height between 1.6m and 2.0m to account for various heights of the occupants. Subsequently, all cell values of the threshold criterion within the volume are averaged. This smoothes the data, which can be very noisy due to physical and numerical issues, to a certain evaluable level. Additional smoothing can be performed by using a time low pass filter like moving averages (Bronstein, 2008). The optical density is recorded centrally in the compartment as people need orientation here. The toxicity levels are recorded near the exits. This is due to the fact that the toxicity levels will be reached later than the visibility threshold (Albrecht, 2009) and people are expected to be near or at least in the vicinity of the exits by then.

The RSET limit from the evacuation simulation is much easier to find: the RSET is said to be reached when the last occupant has left the compartment.

Fire Scenarios

It is not obvious right away which fire scenario will have the greatest effect on the occupants and thus the highest probability of failure. As many scenarios are deemed possible, they can also be regarded along with their probability of occurrence. This allows the consideration of very conservative fire scenarios along with their occurrence probability. For this paper, two important scenarios will be considered exemplary: an ultra-fast fire at the DJ turntables, located south on the dance floor and a fire developing in the bar area, spreading across the shelves and the bar equipment. Both fires follow the traditional tsquared fire scenario with the addition that for the coatroom fire, an incubation phase with a low HRR (~30 kW) is added before the quadratic growth phase is reached. This is in accordance with many fire tests performed as it can be seen in Peacock (1999).

The scenarios along with their outcome in terms of reliability will be modeled in an event tree. The event tree also accounts for the fact that a hostile fire is not a usual event for a building and that some developing fires may be suppressed by staff or occupants and will not develop into a dangerous stage.

Various other scenarios can also be considered for this building but are omitted here for the sake of simplicity. Further information on scenario modeling can be obtained e.g. from NFPA 101 (2008) which requires the successful verification of eight different scenarios for performance-based life safety analysis.

System Components

The two scenarios above are considered without any fire protection systems installed. In order to account for such systems, they have to be modeled within the fire scenarios; either directly within the simulation such as smoke and heat extraction system or within the input parameters or fire scenarios. For this paper, we will show the effect of an automatic detection and alarm system which detects the fire quickly after ignition and alerts the occupants by an audible alarm. This is modeled herein by reducing the premovement time to the shorter time span shown in tab.1.

As such systems-or "fire protection barriers" (Yung, 2008) are prone to failure to "work as designed on demand", this also has to be modeled within the event tree by looking at the two scenarios "functional" and "non-functional" and their outcome along with the probability of failure of the system installed (compare fig. 1). According to BS7974 (2001), automatic detection and alarm system have a reliability to "work as designed on demand" of 90%, and thus a failure probability of 10%, respectively.

Sensitivity Analysis

A sensitivity analysis between the input and the output of both models was performed as described above on a preliminary scan of the random space which required approximately 25 simulations in order to produce accurate results. These calculated "support points" will be re-used for the surrogate construction of the reliability algorithm.

For the optical density, the time to 1 MW (t_g) on the fire side and the occupant density or the resulting number of occupants, respectively, and the premovement time on the evacuation side have the highest influences. The soot yield has a statistically significant yet subordinate impact on the limit state. For the incapacitation, the influence of t_g is even larger and while the influence of both, number of occupants and pre-movement time decreases.



Figure 3: Sensitivities of the fire scenario in the bar area.

Instead of the soot yield, the HCN yield now plays a statistically significant (yet also subordinate) rule. This is due to the fact, that the FED is mainly controlled by the HCN intoxication, as also found by Purser (2002). It should be noted again that the HCN yield was chosen very conservative herein. The results for the bar fire can be seen in fig. 3. The analysis for the ultra fast fire yielded similar results.

Reliability analyses

The reliability analyses were carried out using the approach outlined in the first sections of this paper. In total, approximately 35 fire and evacuation simulations had to be carried out to obtain a sufficient number of support points to derive a probability of failure for one scenario. In order to look at another threshold in the same scenario, the preliminary support points of the first iteration can be re-used so that only eight additional support point simulations are needed for the failure probability of the other threshold criterion.

As the simulations can be executed without any interaction they were processed with a high throughput computer (HTC) which allowed for an fast and efficient computation by load distribution.

The failure probabilities of the two scenarios are summarized in tab. 2.

Table 2: Per scenario and per annum failure probabilities.

Scenario	$\hat{p}_{_f}$	$\hat{p}_{f m_f i}$
Bar fire – Visibility	0.6819	0.0034
Bar fire – FED	0.0540	0.0003
Ultra fast – Visibility	0.9946	0.0050
Ultra fast – FED	0.1429	0.0007

These failure probabilities \hat{p}_f seem extremely high, especially for the visibility criterion. Yet, it should be mentioned again that the latter criterion is reached if the optical density at around 2 m (conservative eye level) falls below a corresponding visibility of 10 m in such a small volume with rather low ceiling heights (4 m) and rather conservative stochastic input parameters. The FED is controlled by the high yields of HCN which were also chosen rather conservative.

Per annum reliability

Additionally, it has to be considered that these probabilities are to be defined as "per hostile fire"-probabilities. In order to use the conventional "per annum"-probability, the frequency of ignition per year and the success rate of intervention have to be accounted for.

The "arrival" of fires is considered random and independent of each other, so that the time in

between fires can be modeled as exponential distributed. Occupancy-specific fire initiation frequencies can be found in the literature, such as BS7974 (2001). For the sake of simplicity, we chose an annual probability of a fire start to be 2%. The same source also gives probabilities of around 50% for the success or failure of manual intervention, respectively. Only regarding those two scenarios, they each have a 50% probability of occurrence if a fire starts and manual intervention fails.

Assuming mutual independence, the probabilities can all be multiplied to derive annual probabilities of failure which are also shown in tab. 2 as $\hat{p}_{f|m_e|i}$, i.e.

annual probability of failure in case of fire initiation and failure of manual intervention.

Summing up the values for each criterion (see section above), both criteria are obviously less than 1%, which implies a probability of occurrence less than once in 100 years on average. This should be regarded in the context of rather conservative and enveloping scenarios and input parameters and assuming very high occupancy at the time the fire occurs.

Consideration of Fire Protection Systems

For the sake of simplicity, the impact of a smoke detection system with subsequent audible alarm at a 10% failure probability is assumed for the bar fire scenario and is modeled into an event tree as shown in fig. 1. The system is modeled by reducing the pre-movement time as described above. The placement of "numerical" smoke detectors within the CFD simulation could also yield a new stochastic model for the warning time.

As the smoke detection and alarm system has no direct influence on the fire development and thus on the simulation, the support point simulations from the previous example can be re-used which saves a great share of computational cost. The evacuation simulations have to re-run with the new stochastic model for the pre-movement time. In order to further reduce the computational cost, a "global" premovement time was chosen (all agents start at the same time) and simply added to the evacuation simulation results. This approach usually leads to more conservative results as all agents evacuate at the same time leading to higher utilization of the exits per unit time. Additionally, this has the positive effect that the support point simulations for FDS+evac can also be re-used and only very few new simulations are required for the additional surrogate iterations.

The results of the reliability analyses for the visibility and the FED criteria are shown in tab. 3. Table 3: Results of the reliability analyses with
smoke detection and alarm system
compared to the case without.

Scenario	$\hat{p}_{_f}$
Visibility with detection & alarm	0.2142
Visibility w/o detection & alarm	0.6819
FED with detection & alarm	0.0174
FED w/o detection & alarm	0.0540

Considering the 10% failure probability for the detection and alarm system, the per hostile fire probability of failures of a save egress for the criteria are as follows:

- Optical density: 0.2610
- FED: 0.0211,

which is a reduction of the failure probability by a factor greater than two for both criteria-even though the failure of the installed system is already considered.

In order to derive the relevant per annum probabilities, the system analysis can be performed as described above, leading to the very low failure probabilities $\hat{p}_{f|m_{e}|i}$ of:

- Optical density: 13.0x10⁻⁴
- FED: 1.05x10⁻⁴

CONCLUSIONS & OUTLOOK

With the response surface methodology described in the first part of this paper, we enable the reliability analysis of life safety design using state-of-the-art fire engineering tools without the unbearable computational costs of Monte Carlo simulation and without the large approximation errors of traditional least-square response surface modeling.

The methodology was exemplary applied to a generic problem in Fire Protection Engineering and proved to work accurate and fast. In order to perform the analyses, various stochastic models had to be found, mainly based on literature reviews and educated assumptions, as detailed statistical information is currently unavailable. Hence, the calculated failure probabilities should be regarded as "operational" failure probabilities, as they are highly dependent on the models, scenarios, and input parameter distributions chosen. As these factors are deemed rather conservative, the safety level is likely to be higher.

Using event tree system analyses and a very simplified model to consider the effect of a smoke detection and alarm system, a quantitative impact could be derived.

The results can be used for a quantitative indicator of the safety level and various designs can be compared for their safety level, as long the same underlying models, parameters etc. are used. Additionally, the designer can compare various strategies and/or fire protection systems and objectively find the most costbenefit-effective solution without the subjective "gutfeeling".

In further research work, various other fire protection systems will be modeled and analyzed for their quantitative impact on the life safety level. Incorporating potential costs for the system and for the expected impact on the occupants, a fully quantitative risk analysis can be performed.

The most important next steps will be the collection and derivation of sufficient stochastic models for the various parameters along with establishing acceptable reliability requirements, i.e. by calibration using various examples which are compliant with currently accepted "deemed-to-satisfy" prescriptive codes and regulations.

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