STOCHASTIC ANALYSIS OF EGRESS SIMULATIONS

JULLIEN Quentin and LARDET Paul

[1] Université Paris-Est, Centre Scientifique et Technique du Bâtiment (CSTB), Safety, Structures & Fire Department, Expertise, Regulatory Advice & Research Division
quentin.jullien@cstb.fr
[2] Université Paris-Est, Centre Scientifique et Technique du Bâtiment (CSTB), Safety, Structures & Fire Department, Expertise, Regulatory Advice & Research Division
lardet.paul@cstb.fr

Abstract. The use of egress simulation models in performance-based analysis relies on the confidence in the input and output data. These data strongly depend on a large number of parameters. The study presented here focuses on the statistical aspects of these data, especially the population behavioral parameters which are the most likely to be scattered. The study proposes a method to analyze the statistical aspects of an egress simulation model. The method is based on statistical estimations of the distribution quantiles of the output parameters and can be applied to stochastic simulations results. It provides quantitative informations on the key output parameters dispersion, such as Required Safe Egress Time (RSET). It also gives a justification to the required number of simulations and input parameters precision to ensure a relevant output precision level. It gives access to quantitative criteria to compare experimental and numerical data. This method will be applied to analyze case studies simulated with BuildingExodus.

1. INTRODUCTION

The practice of the egress engineering relies on numerical models. Their main purpose is to determine the time taken by last occupant to evacuate a building either in normal or accidental situation. This time is called Required Safe Egress Time (RSET) all along the present document. The various human behavior phenomena occurring during an evacuation are not deterministic. Indeed, the intrinsic behavior of a person, for example its response time before movement, can be scattered from case to case. As a result, the egress phenomena is scattered in itself, and a given starting situation can lead to many different evacuations process.

Some previous work has already dealt with this scattered aspect of the phenomena. For example, Tavares and Ronchi [2] propose a method to evaluate and take into account uncertainties in egress engineering studies. Cuesta, Ronchi and Gwynne study some school building evacuation test data [1]. They then use those data as inputs for numerical simulations of the egress process and propose a comparison method between experimental and numerical results.

Although these previous work cover the scattered aspects of one particular situation, it is still quite difficult to encompass the whole range of possibilities of an egress process. The work presented here proposes a method to study these different possibilities. This method includes the scattered aspects of the human behavior, as well as the different usages for a given building. A stochastic approach is set up for that purpose, in order to evaluate a relevant RSET value. A statistic treatment is also applied in order to take into account the random aspects of the resulting data. Using a numerical tool is appropriate as it allows to easily perform a large number of RSET evaluations.

The numerical tool used in this research study is BuildingExodus v6.2. It uses sophisticated sub-models means to take into account interactions like occupant-occupant, occupant-fire and occupant-building. Each occupant is individually modeled and has its own characteristics. This is essential to the study presented here. BuildingExodus is also able to process the large number of simulations needed to conduct this study.

The statistical analysis method set up here requires some precautions. First, it is essential to identify the parameters (response time, walking speed, interactions between people), which may impact the outputs. The statistical distribution of their value is needed in order to produce relevant results. However, very few statistical informations are available from literature. Usually only a minimum, maximum and a mean values are available. Thus, only uniform distribution laws are used in this study. This first order hypothesis does not impact the mathematical method, but needs further improvements.

The statistical analysis method is based on statistical estimation of the output parameters quantile distribution (RSET in this document). The aim of this method is to provide quantitative elements on statistical distribution of RSET. It gives quantitative arguments to validate the number of simulations performed, compared to the expected precision. A particular focus will be done on the RSET 95^{th} percentile.

2. STATISTICAL ANALYSIS METHOD: IMPLEMENTATION AND STATISTICAL ELEMENTS

As an example, the statistical analysis method is applied to a sample of n realizations of a random variable which distribution is normal. In the following sections the variable considered will be the RSET. 60 samples are randomly drawn.

A confidence interval $\mathrm{I_p}$ is calculated for each quantile α from the sample of size n = 60 by choosing a level of confidence p equal to 90%. The confidence interval is the interval which includes the real value of the estimated quantiles with a 90% probability. This confidence interval is calculated as follows:

$$Ip = \left[\tilde{F}^{-1}\left(\alpha - c\sqrt{\frac{\alpha(1-\alpha)}{n}}\right); \tilde{F}^{-1}\left(\alpha + c\sqrt{\frac{\alpha(1-\alpha)}{n}}\right)\right]$$

with:

- $\tilde{F}$ the empirical distribution function,

– c the quantile $(1 - \frac{p}{2})$ of the normal law distribution. c is equal to 1.645 for p = 90%.

The distribution function is said empirical as opposed to the real distribution function, which is generally unknown.

Figure 1 shows the random variable realizations ranked in ascending order generating the empirical distribution function as well as lower and upper bounds of each percentile confidence intervals.

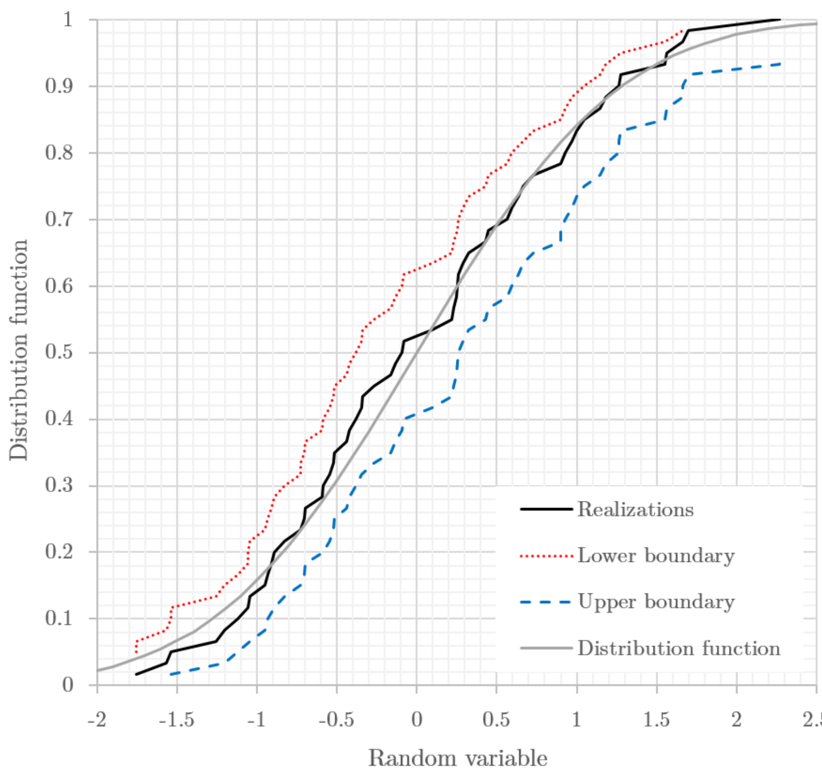

Figure 1. Empirical distribution function of the random variable, lower and upper bounds of the confidence interval and real distribution function

This example highlights the problems related to extreme values of a random variable: in some cases, the random variable will have no maximum upper value. This is the case in this simple example, but can also happen for some RSET evaluations.

In most of the cases, a maximum value exists, but the finite samples number prevent from calculating this maximum value. A stochastic analysis can indeed only give percentiles, which get closer to 100% when the number of sample grows.

In addition, a maximum RSET value can reflect very extreme scenarios where all worst case situations happen simultaneously. The value of this kind of extreme cases is questionable.

Furthermore, the confidence interval width decreases when the samples number increases.

Consequently, the required number of draws required by a stochastic analysis is set by:

- the order of the desired percentile,
- the required precision, which imposes the size of the confidence interval.

3. HYPOTHESIS OF THE MODEL

The numerical tool used in this study is BuildingExodus. However, any kind of egress simulation could be processed by the mean of the proposed method.

BuildingExodus is based on a discretization of space by interconnected nodes of 50 cm x 50 cm. The connection model is the Moore's model as shown on figure 2.

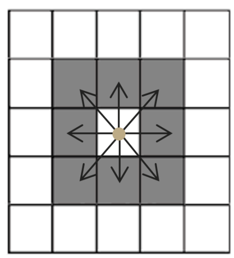

Figure 2. Nodes connectivity under BuildingExodus

The test case is a 16 m square room with 4 exits distributed on each side (figure 3). Each occupant occupies one complete node. Two occupants cannot coexist on the same node. The exits are 3 m of wide, and their flow rate is 2.0 occupant/m/s.

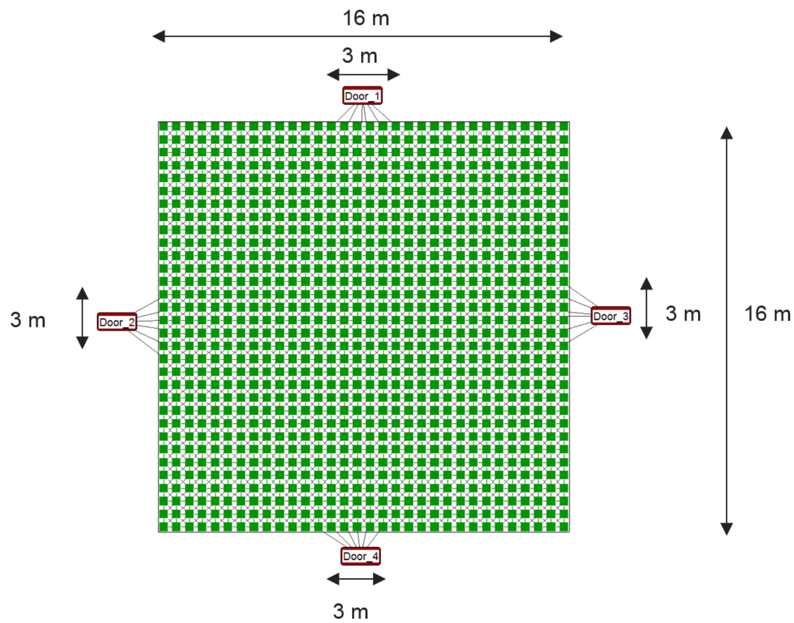

Figure 3. Test case geometry

Concerning population, some attributes are arbitrarily fixed and others are variable depending on test cases:

- occupants have identical leaderships (in this case 10). This parameter affects how conflicts are resolved when two occupants want to occupy the same node which is impossible. BuildingExodus applies by default a conflict resolution time between 0.8 s and 1.5 s to the occupants. For some test cases, the conflict resolution time is set at the average value of the interval, that is to say 1.15 s.
- occupant's patience is imposed to 10 s. This implies that the occupants are willing to remain static in a queue for 10 seconds before attempting to change direction.
- occupants are all valid,
- their speed is 1.2 m/s,
- the response time is set at 15 s or variable between 0 and 30 s depending on the test case. The response time in this study is the delay before the occupants begin to move toward the exit.
- occupants act independently of each other: if an occupant begins to move toward the exit he will not make another occupant move before its response time is elapsed.

This document do not present the attributes specifically related to fire such as incapacitating concentrations of toxic gases (HCl, HBr, HF, SO_2, NO_2, CH_2CHO,...) because they have no influence on the treated cases.

Varying parameters are randomly drawn according to a uniform law between two extreme values. The most advanced behavioral options are left at their default values (e.g: occupants are aware of all exits). The objective of the occupants is to reach the nearest exit from their initial position. Finally, when several simulations in large numbers for the same test cases are performed, they are not used again from one case to another. That is to say that if 100 and 1,000 achievements are performed, the first 100 are not part of the following 1,000. The draw is completely redone every time. These choices are made for simplification purposes. They are meaningless regarding to the proposed analysis method.

4. STUDY

4.1. Use of the statistical analysis method

In this reference test case, two random simulations sets are carried out. Each simulation takes into account a random occupant number between 1 and 1,000, randomly located in the room. The first set contains 100 simulations, the second one contains 1,000 simulations. Figures 4 and 5 show the calculated RSET distribution functions as well as the lower and upper bounds of the confidence intervals for 100 simulations, respectively 1,000 simulations. The selected level of confidence is 90% (the same level of confidence is used in the rest of the document).

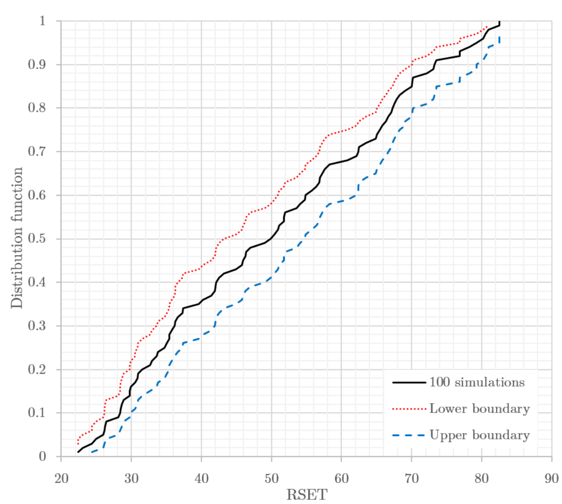

Figure 4. Distribution function for 100 simulations and lower and upper bounds of the confidence interval (reference test case)

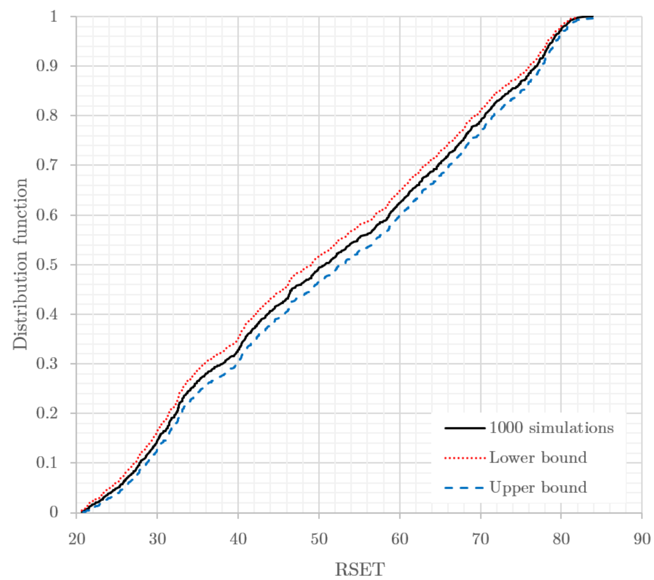

Figure 5. Distribution function for 1,000 simulations and lower and upper bounds of the confidence interval (reference test case)

Note that the number of simulations is too low to reach the theoretical minimum RSET corresponding to a single occupant positioned in front of an exit. Indeed, one could expect a RSET close to 15 s in this case (RSET = response time + traveling time, minimum traveling time being 0 s).

In accordance with what has been announced above, it is observed that:

- the confidence interval width decreases when simulation number increase from 8.1 s on average for 100 simulations to 2.5 s on average for 1,000 simulations,
- extreme percentiles have unbound confidence intervals for 100 and 1,000 simulations.

Thus, it is not possible to statistically determine a maximum RSET value. Table 1 shows the confidence intervals associated with the 95$^{\text{th}}$ percentile for 100, 200, 500, 1,000 and 5,000 simulations. This choice implies that in 95% of cases all occupants have evacuated with a probability of 90%. The choice of the studied percentile order should be discussed in further studies, as it is the key parameter associated to the building safety level. The confidence level impacts the confidence interval width, and must be chosen according to the required precision (see section 2).

	Confidence interval of the 95$^{\text{th}}$ percentile (s)	Width of confidence interval of the 95$^{\text{th}}$ percentile (s)
100 simulations	[76.9 ; 82.6]	5.7
200 simulations	[76.6 ; 80.0]	3.4
500 simulations	[78.0 ; 79.7]	1.7
1,000 simulations	[78.4 ; 79.3]	0.9
5,000 simulations	[78.1 ; 78.8]	0.7

Table 1. Confidence intervals of the 95$^{\text{th}}$ percentile for 100, 200, 500, 1,000 and 5,000 simulations

Figure 6 shows the confidence interval width for the 95$^{\text{th}}$ percentile according to the number of simulations. There are a number of simulations for which the width of the confidence interval is sufficiently low to be acceptable. Moreover, beyond this number an increase of the number of simulation does not provide significant accuracy (see table 2) while it considerably increases the computing time. Indeed, the simulations are all performed in very short times, despite the fact that the population can vary from 1 to 1,000 occupants: the time spent to perform the simulations is proportional to the number of simulations (simulations are achieved in a sequentially way). So for all these reasons, 1,000 simulations seem to be sufficient in the case studied here.

In conclusion, the proposed statistical analysis method quantitatively evaluates the number of simulations that seems most relevant to carry out a study.

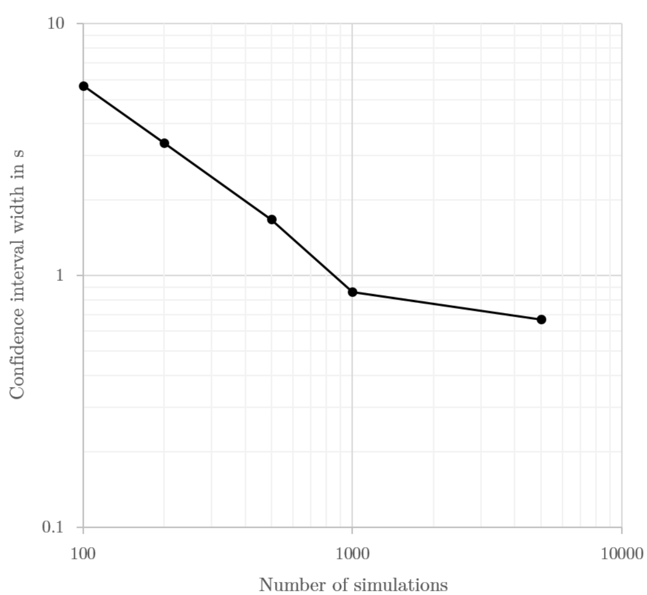

Figure 6. Confidence interval width vs. number of simulations

	Decrease of the confidence interval width (%)
100 simulations	Reference
200 simulations	40
500 simulations	70
1,000 simulations	84
5,000 simulations	88

Table 2. Decrease of the width of the confidence interval according to the number of simulations

4.2. Complementary analysis

At least three separate evacuation patterns exist according to the number of occupants (see figures 4 and 5) the table 3 gives the correspondence with the RSET obtained. These three patterns correspond to three density ranges of people highlighting its influence on the RSET. Indeed, the density of people has influence on the congestion time and the average occupants speed.

	Pattern 1	**Pattern 2**	**Pattern 3**
RSET (s)	[22.4 ; 37.4]	[37.4 ; 72.2]	[72.2 ; 82.6]
Occupants number	[34 ; 297]	[297 ; 806]	[806 ; 982]
Population density (person/m^2)	[0.1 ; 1.2]	[1.2 ; 3.1]	[3.1 ; 3.8]
Average waiting time (s)	2.7	13.5	23.5
Average speed (m/s)	0.8	0.3	0.2

Table 3. Ranges of differents patterns for 100 simulations

The figures 7 and 8 show the occupants number scattering according to RSET for 100 and 1,000 simulations. A greater dispersion is observed for the pattern n°1. In this case, the RSET is controlled by the distance from the last occupant to the exit, rather than the population density. It demonstrates the interest of the approach developed in this document, which provides additional elements to understand evacuation behaviors.

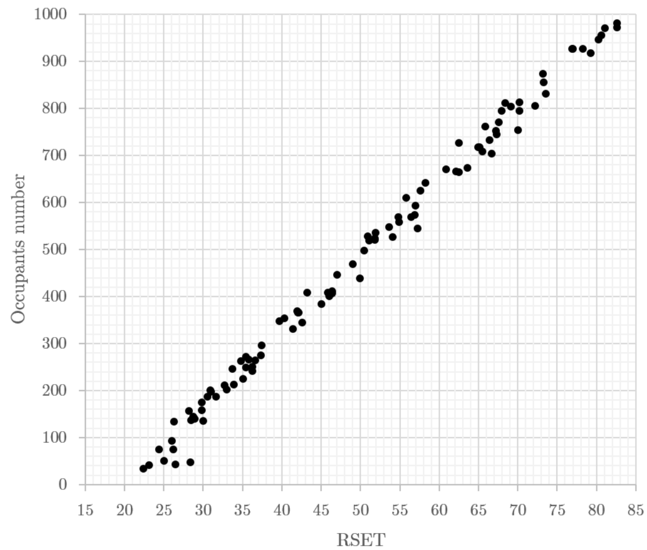

Figure 7. Occupants number scattering vs. RSET for 100 simulations

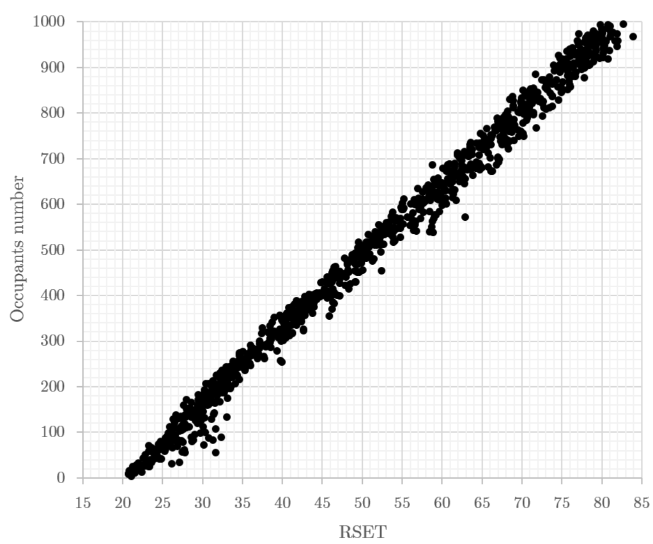

Figure 8. Occupants number scattering vs. RSET for 1,000 simulations

As there are very little simulations with the same number of occupant, it is impossible to quantify the influence of position and conflict resolution time on the RSET at this stage of the study. This is the topic of the next section.

5. ANALYSIS OF THE PARAMETERS' STATISTICAL INFLUENCE

This section studies the influence of some parameters on the results of the reference test case for the three patterns previously identified. Table 4 shows the characteristics of each test case.

	Parameters of the sensibility study			
	Conflict resolution time (s)	**Position of the occupants**	**Response time (s)**	**Number of simulations**
Test case 1	[0.8 ; 1.5]	Fixed	15	
Test case 2	1.15	Fixed	15	1000
Test case 3	[0.8 ; 1.5]	Random	15	
Test case 4	[0.8 ; 1.5]	Random	[0 ; 30]	

Table 4. Synthesis of studied influences for the test cases n°1, 2, 3 and 4

For each case 3 sets of simulations are performed. Each of these sets takes into account a fixed number of people, corresponding to the median RSET of each pattern identified above.

Table 5 shows the characteristics of these simulations.

	pattern 1	**pattern 2**	**pattern 3**
Reference case RSET (s)	30.5	55.8	78.3
Occupants number	187	610	927
Population density (person/m^2)	0.7	2.4	3.6

Table 5. Simulations selected for test cases n°1, 2, 3 and 4

5.1. Test case n°1

Three sets of 1,000 simulations are run, each set having a fixed occupant number and occupant location. The conflict resolution time is the only random parameter, and has a uniform distribution law. Figure 9, 10 and 11 present the distribution functions as well as the lower and upper bounds of the confidence intervals. The empirical distribution functions are clearly not uniform. Yet, only the conflict resolution time is scattered. It demonstrates the sophisticated interactions between input parameters and RSET, as a uniform random parameter can lead to complex statistical REST behavior.

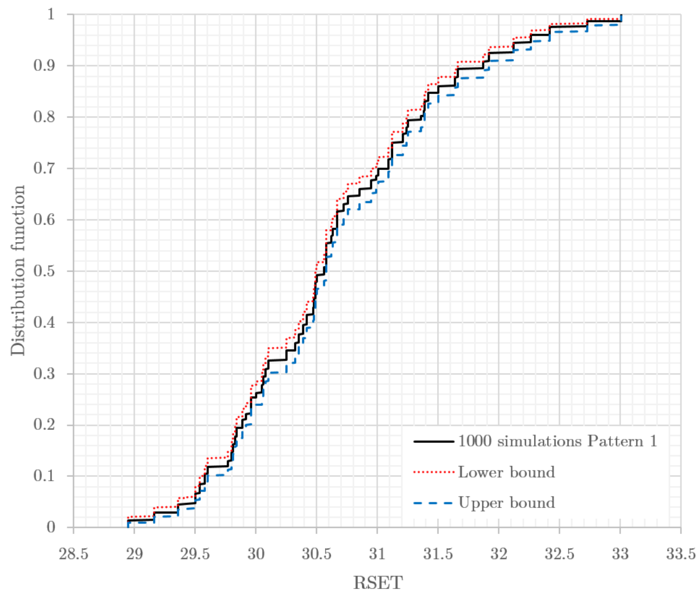

Figure 9. Distribution function for 1,000 simulations and lower and upper bounds of the confidence interval for pattern 1 (test case n°1)

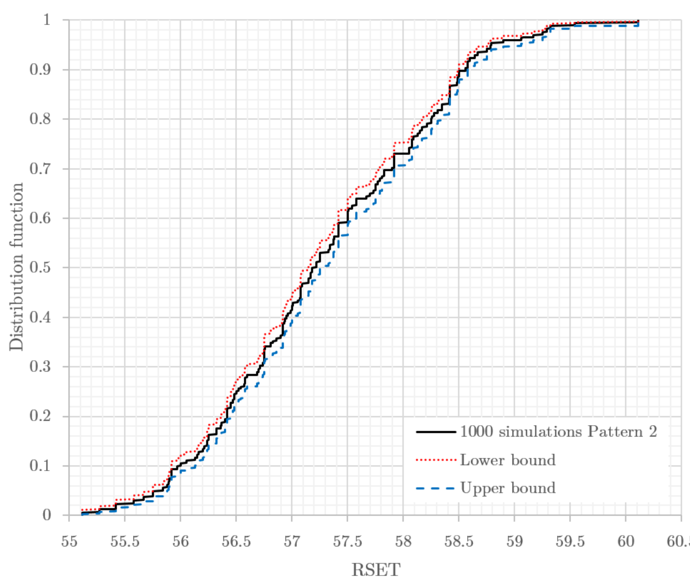

Figure 10. Distribution function for 1,000 simulations and lower and upper bounds of the confidence interval for pattern 2 (test case n°1)

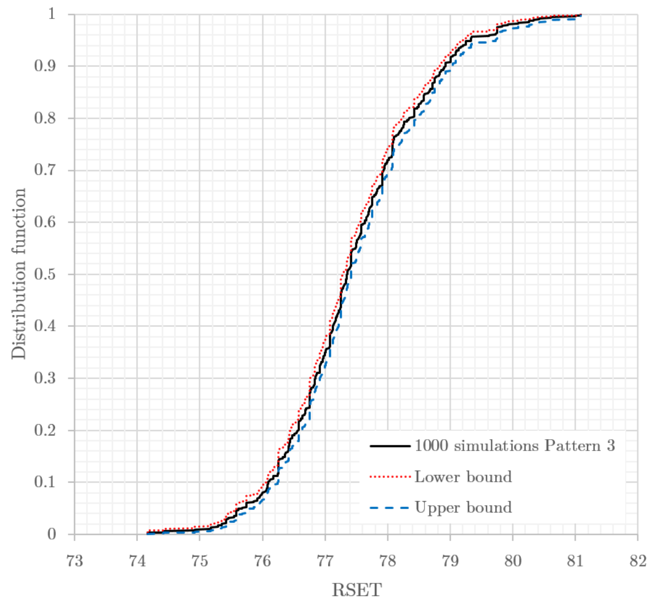

Figure 11. Distribution function for 1,000 simulations and lower and upper bounds of the confidence interval for pattern 3 (test case n°1)

Table 6 summarize the three simulations sets results:

- 95^{th} percentile confidence interval ($I_{p95\%}$),
- width of $I_{p95\%}$ $L_{Ip95\%}$ (see figure 12),
- the interval between the lower bound of the 5^{th} percentile and the upper bound of the 95^{th} percentile $I_{p5\%-95\%}$,
- width of $I_{p5\%-95\%}$ $L_{Ip5\%-95\%}$ (see figure 12),
- the ratio between $L_{Ip5\%-95\%}$ and the 50^{th} percentile $L_{Ip5\%-95\%}/q_{50\%}$.

	RSET reference test case (s)	$\mathbf{I_{p95\%}}$ **(s)**	$\mathbf{L_{Ip95\%}}$ **(s)**	$\mathbf{I_{p5\%-95\%}}$ **(s)**	$\mathbf{L_{Ip5\%-95\%}}$ **(s)**	$\mathbf{L_{Ip5\%-95\%}/q_{50\%}}$ **(%)**
S. 1	30.5	[32.1 ; 32.4]	0.3	[29.4 ; 32.4]	3.0	10.0
S. 2	55.8	[58.8 ; 59.1]	0.3	[55.8 ; 59.1]	3.3	5.8
S. 3	78.3	[79.2 ; 79.7]	0.5	[75.6 ; 79.7]	4.1	5.4

Table 6. Synthesis of the results obtained

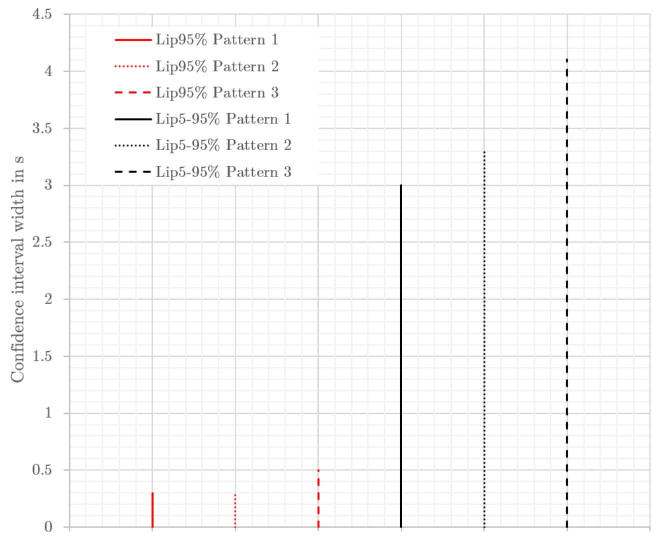

Figure 12. Confidence intervals width for the 3 patterns of the test case n°1

There is a certain variability of the RSET even when the starting positions of the occupants are fixed. As commented above, this variability is due to the variability of the conflict resolution time as well as to the effect of history produced by the various collisions. It is coherent with the fact that $L_{Ip95\%}$ is higher for the pattern including a bigger density of people. The same phenomenon is observed for $L_{Ip5\%-95\%}$. It is reminded that the choice of the 95th percentile and by extension the one of the 5th percentile depends on the objectives of the study.

5.2. Test case n°2

For this second test case the conflict resolution time is fixed to its average value 1.15 s. This allows to separate the impact of the conflict resolution time variability from the history effect. Again, 3 sets of 1,000 simulations are run, each one with a fixed occupant number and occupant location.

In this case too, the distribution functions are not uniform despite the fact that conflict resolution time is fixed to 1.15 s. It demonstrates the strong non-linearity of the history effects. In addition, this implies that the distribution functions of test case 3 and 4 will not be uniform. Synthetically, table 7 presents $I_{p95\%}$, $L_{Ip95\%}$, $I_{p5\%-95\%}$, $L_{Ip5\%-95\%}$ and $L_{Ip5\%-95\%}/q_{50\%}$ for the three simulations and compared to those of the test case n°1. The widths of intervals are included in figure 13. This demonstrates the very weak influence of the conflict resolution time on the results, at least in this simple case.

Test case	S.	$\mathbf{I_{p95\%}}$ (s)	$\mathbf{L_{Ip95\%}}$ (s)	$\mathbf{I_{p5\%-95\%}}$ (s)	$\mathbf{L_{Ip5\%-95\%}}$ (s)	$\mathbf{L_{Ip5\%-95\%}/q_{50\%}}$ (%)
	1	[32.1 ; 32.4]	0.3	[29.4 ; 32.4]	3.0	10.0
1	2	[58.8 ; 59.1]	0.3	[55.8 ; 59.1]	3.3	5.8
	3	[79.2 ; 79.7]	0.5	[75.6 ; 79.7]	4.1	5.4
	1	[31.0 ; 32.0]	1.0	[29.2 ; 32.0]	2.8	9.2
2	2	[58.8 ; 58.9]	0.1	[56.0 ; 58.9]	2.9	5.1
	3	[79.3 ; 79.8]	0.5	[75.4 ; 79.8]	4.4	5.6

Table 7. N°2 test case results synthesis and comparison with test case n°1

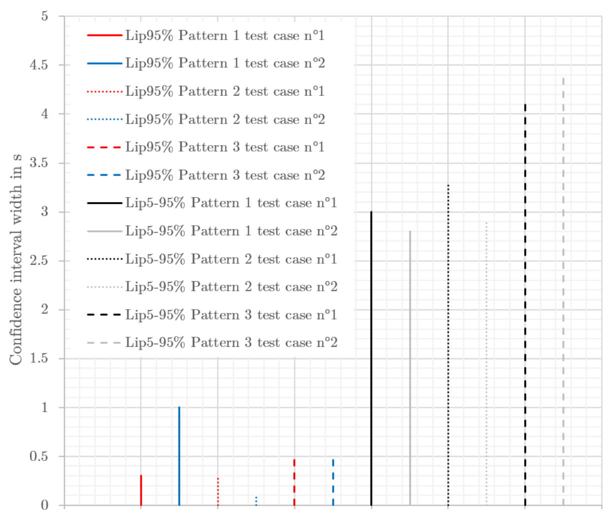

Figure 13. Confidence interval width comparison for the 3 patterns between test cases n°1 and n°2

5.3. Test case n°3

For this third test, conflict resolution time and occupants position are randomly drawn. Again, 3 sets of 1,000 simulations are run, each one with a fixed occupant number.

Table 8 summarize the three simulations sets results and compares them to n°1 test case. The widths of interval are included in figure 14. It quantifies the influence of position. As could be expected,its influence is greater on the low and average densities of population than on a strong density.

Test case	S.	$I_{p95\%}$ (s)	$L_{Ip95\%}$ (s)	$I_{p5\%-95\%}$ (s)	$L_{Ip5\%-95\%}$ (s)	$L_{Ip5\%-95\%}/q_{50\%}$ (%)
1	1	[32.1 ; 32.4]	0.3	[29.4 ; 32.4]	3.0	10.0
	2	[58.8 ; 59.1]	0.3	[55.8 ; 59.1]	3.3	5.8
	3	[79.2 ; 79.7]	0.5	[75.6 ; 79.7]	4.1	5.4
3	1	[33.3 ; 33.6]	0.3	[29.6 ; 33.6]	4.0	13.0
	2	[61.4 ; 62.1]	0.7	[55.7 ; 62.1]	6.4	11.0
	3	[79.8 ; 80.2]	0.4	[75.2 ; 80.2]	5.0	6.4

Table 8. N°3 test case results synthesis and comparison with n°1

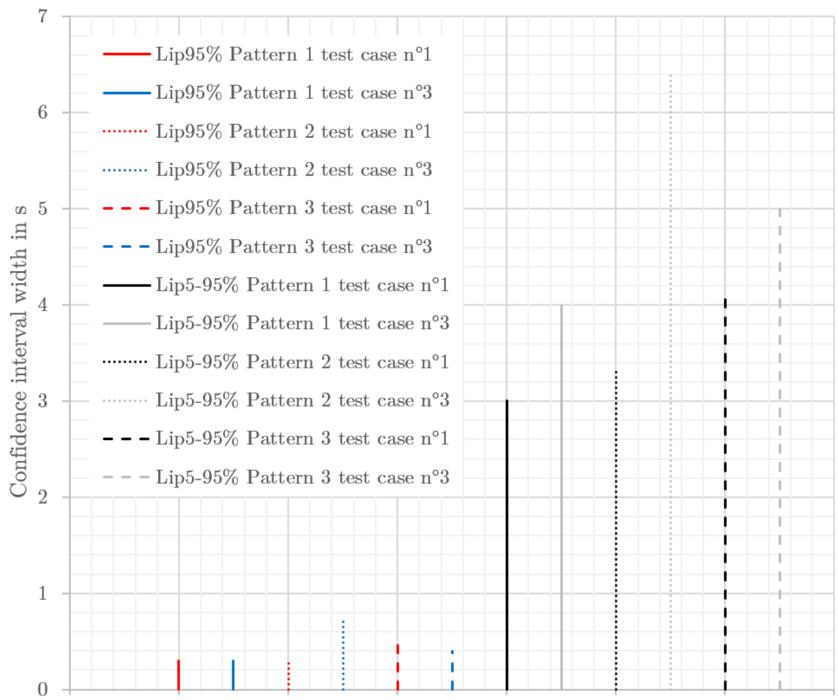

Figure 14. Confidence interval width comparison for the 3 patterns between test cases n°1 and n°3

5.4. Test case n°4

For this fourth test case, conflict resolution time, occupant location and response time are randomly drawn. Again, 3 sets of 1,000 simulations are run, each one with a fixed occupant number.

Table 9 summarize the three simulations sets results and compared to n°3 test case. The intervals widths are presented in figure 15. The response time variability strongly affects the RSET distributions patterns 2 and 3. The RSET minimum value increases for pattern n°1 when the variability is added to the response time. This may be caused by some few extreme cases drawn among the 1,000 simulations.

Test case	**P.**	$\mathbf{I_{p95\%}}$ **(s)**	$\mathbf{L_{Ip95\%}}$ **(s)**	$\mathbf{I_{p5\%-95\%}}$ **(s)**	$\mathbf{L_{Ip5\%-95\%}}$ **(s)**	$\mathbf{L_{Ip5\%-95\%}/q_{50\%}}$ **(%)**
3	1	[33.3 ; 33.6]	0.3	[29.6 ; 33.6]	4.0	13.0
	2	[61.4 ; 62.1]	0.7	[55.7 ; 62.1]	6.4	11.0
	3	[79.8 ; 80.2]	0.4	[75.2 ; 80.2]	5.0	6.4
4	1	[36.9 ; 37.8]	0.9	[33.9 ; 37.8]	3.9	11.0
	2	[61.4 ; 62.4]	1.0	[50.9 ; 62.4]	11.5	20.7
	3	[82.3 ; 83.1]	0.8	[74.3 ; 83.1]	8.8	11.3

Table 9. N°4 test case result synthesis and comparison with n°3

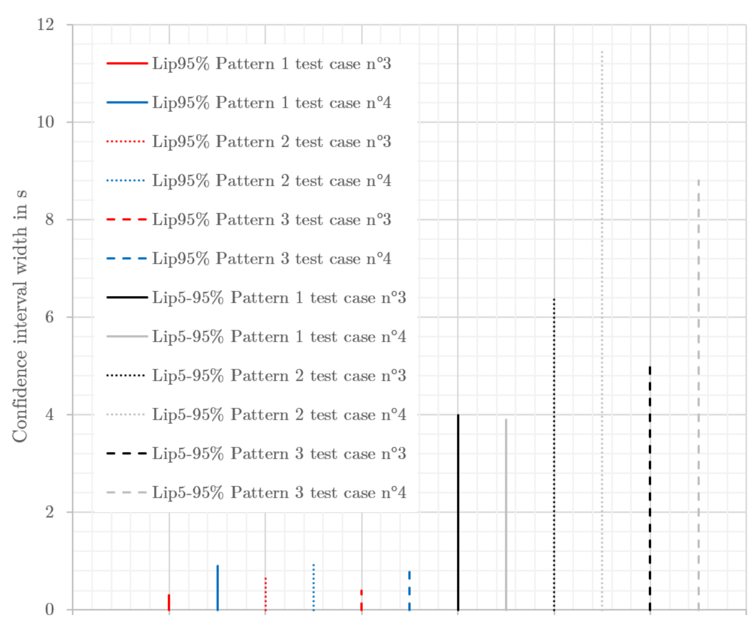

Figure 15. Confidence interval width comparison for the 3 patterns between test cases n°3 and n°4

6. SYNTHESIS

The proposed statistical analysis method has several assets addressing different questions. First, it gives a methodological justification to the RSET quantification. The confidence interval approach allows to evaluate the simulations precision, by explicitly calculating a range of probable values. This cannot be achieved by a unique deterministic approach, which relies on a preliminary worst case evaluation. The stochastic approach also gives elements of proof to evaluate the worst case scenarios. Even in the simple cases studied in this document with uniform distributions, the interactions between parameters generates non-intuitive complex behavior. Therefore, it can be difficult to evaluate which case is worst without quantified analysis. Obviously, the influences of the different parameters depends on their statistical distribution, which is usually poorly known. But using a stochastic approach can show the relative influence between parameters as well as their interactions. Therefore it helps sorting out the main influence parameters on which the experimental efforts should be focused.

Table 10 summarizes the influences of the different parameters tested in these simple cases.

Studied parameters	**Variation range of the input parameter**	**Qualitative influence**
Number of occupant	From 1 to 1,000 persons or 187, 610 and 927 persons	Very important
Position of the occupant	Fixed or random	Important
Conflict resolution time	From 0.8 s to 1.5 s or 1.15 s	Negligible
Response time	From 0 s to 30 s or 15 s	Important

Table 10. Synthesis of the parameters statistical influence study

Of course, the results obtained in this study are preliminary results. They need to be extended to more complex geometries and more realistic parameters dispersions.

The stochastic approach also gives complementary informations on the egress phenomena. As a large number of scenarios can be analyzed, some behavior and interactions are addressed that could not be accessed by a single deterministic approach.

Incidentally, this approach raises the question of what RSET is. Indeed, it shows that a maximum RSET value can be difficult or even impossible to quantify. Therefore, it should be relevant to discuss what could be an adequate failure probability? This question has very large implications, which are linked to the intrinsic scattered nature of the human behavior phenomena.

7. CONCLUSION

The work presented here is a first step towards a statistical view on egress engineering. Is focuses on the quantified results given by the mathematical tools. There is still a lot of work to produce in order to get an engineering level tool.

First, the method developed here has been carried out by simple automation scripts. It should be enhanced and industrialized by using stochastic-dedicated tools.

Furthermore, and maybe above all, the input data required by this method still have to be refined. The input parameters distributions should especially be studied. As presented above, applying a stochastic approach can help sorting out the main influences, and prioritizing which evacuation tests to perform in order to get these informations. This action requires deeper and more extensive analysis of the different behavior parameters, possibly using experimental plans. A comparison between various egress engineering tools parameters should also be carried out.

Finally, the quantitative precision and evaluation criteria given by this approach can also help comparing simulations and experiments. It can also help setting up relevant test protocol in order to maximize the useful informations amount produced by these tests. This kind of work seems achievable once a good knowledge of the input parameters is available.

REFERENCES

[1] Enrico Ronchi, Arturo Cuesta, and Steven M. Gwynne. *Collection and Use of Data from School Egress Trials*. Interscience communications. 2015.

[2] Rodrigo Machado Tavares, and Enrico Ronchi. *Uncertainties in Evacuation Modelling: Current Flaws and Future Improvements*. Interscience communications. 2015.