Direct Calculation of Ellipse Overlap Areas for Force-Based Models of Pedestrian Dynamics

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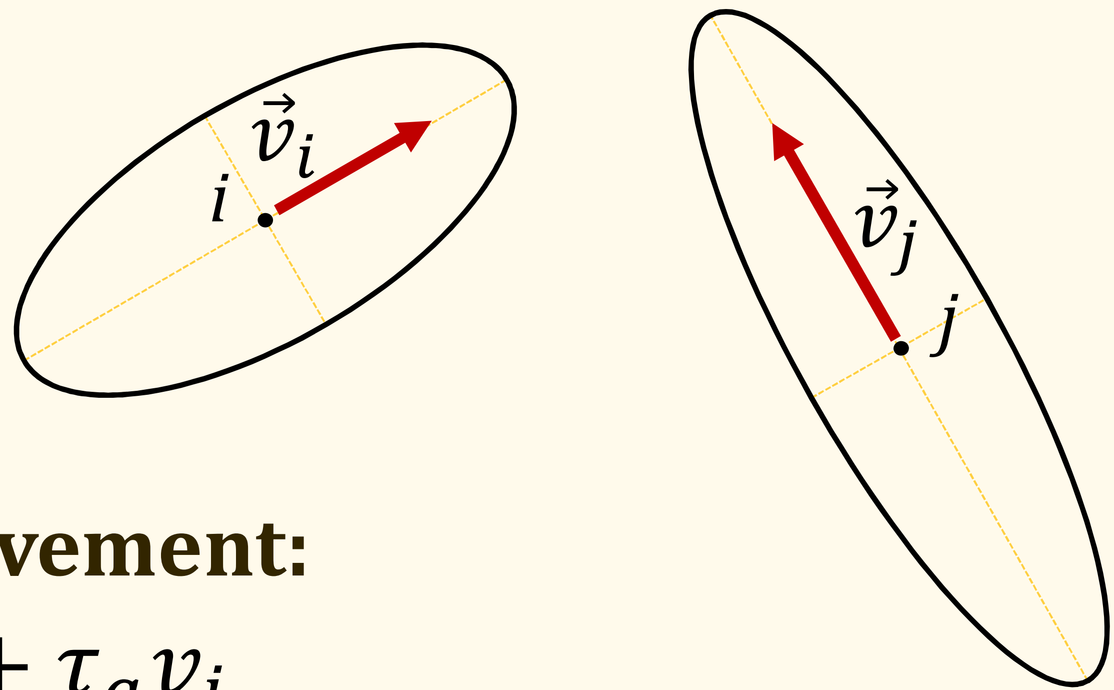
Pedestrian Spatial Aspect

- Radially Asymmetric
- Velocity-Dependent

Dynamic, Elliptical 'Sensory Zone'



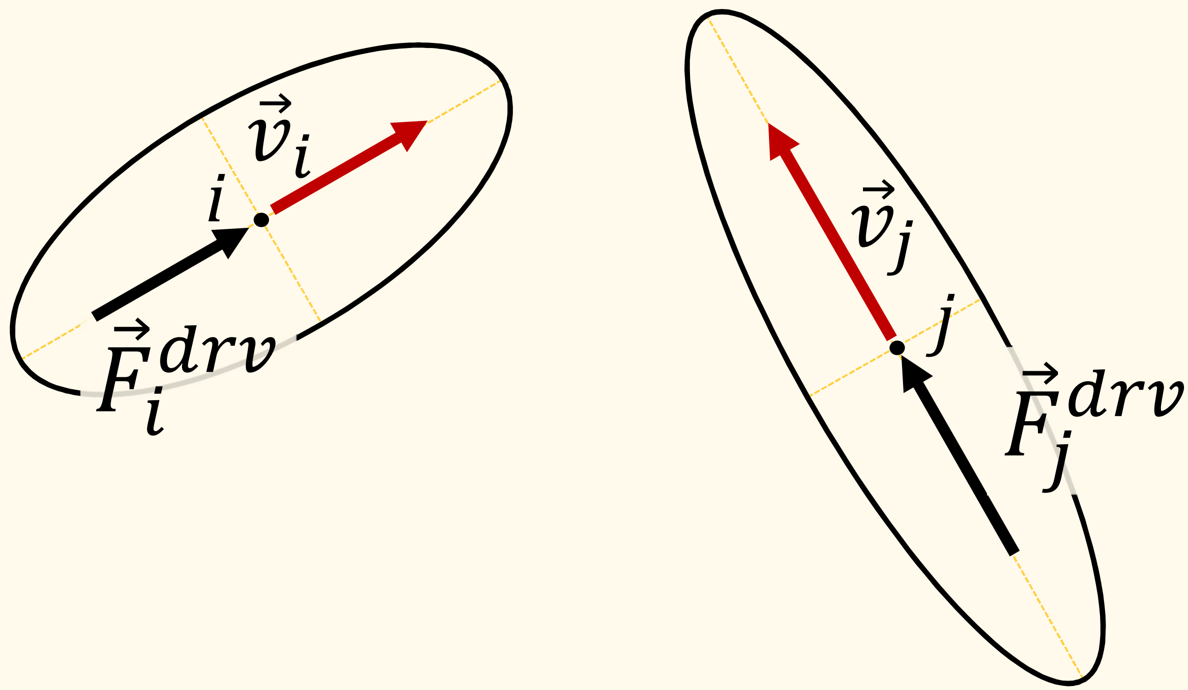
Semi-Axis
Direction of Movement:

$$a = a_{min} + \tau_a v_i$$

Semi-Axis
Lateral Swaying:

$$b = b_{max} - (b_{max} - b_{min}) \frac{v_i}{v_i^0}$$

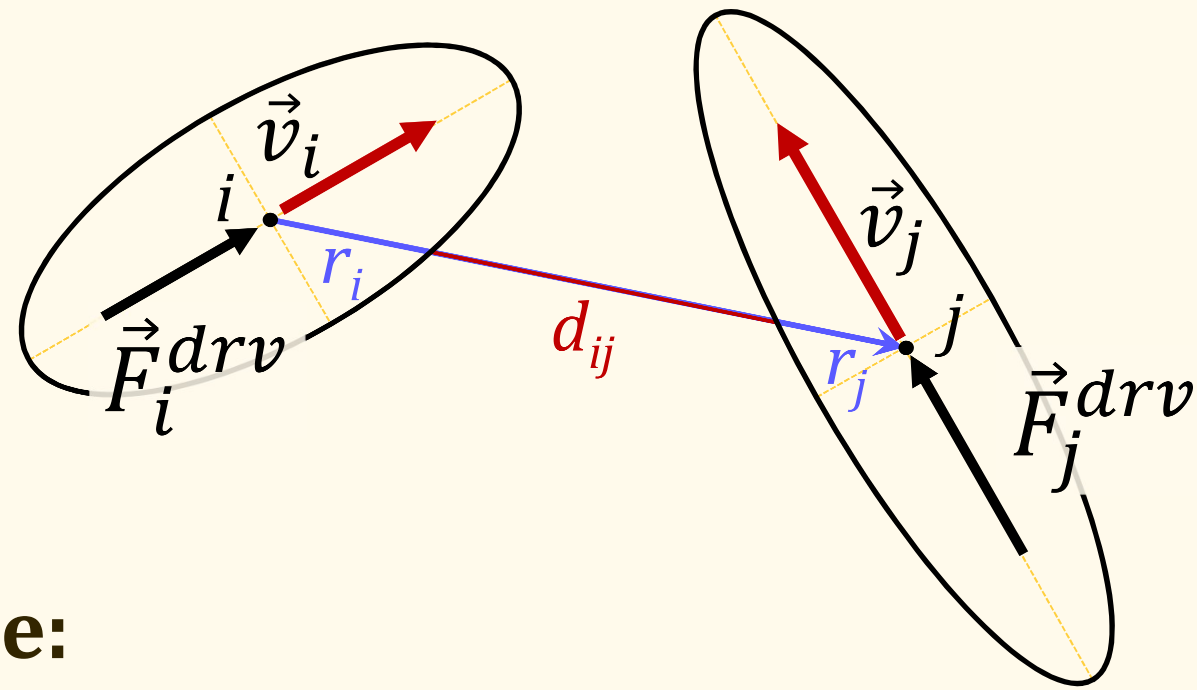
Chraibi, M., Seyfried, A. and Schadschneider, A. (2010), "Generalized centrifugal-force model for pedestrian dynamics," *Physical Review E*, **82**:4, p. 046111.



Driving Force:

$$\vec{F}_i^{drv} = m_i \frac{\vec{v}_i^0 - v_i}{\tau}$$

Chraibi, M., Seyfried, A. and Schadschneider, A. (2010), "Generalized centrifugal-force model for pedestrian dynamics," *Physical Review E*, **82**:4, p. 046111.

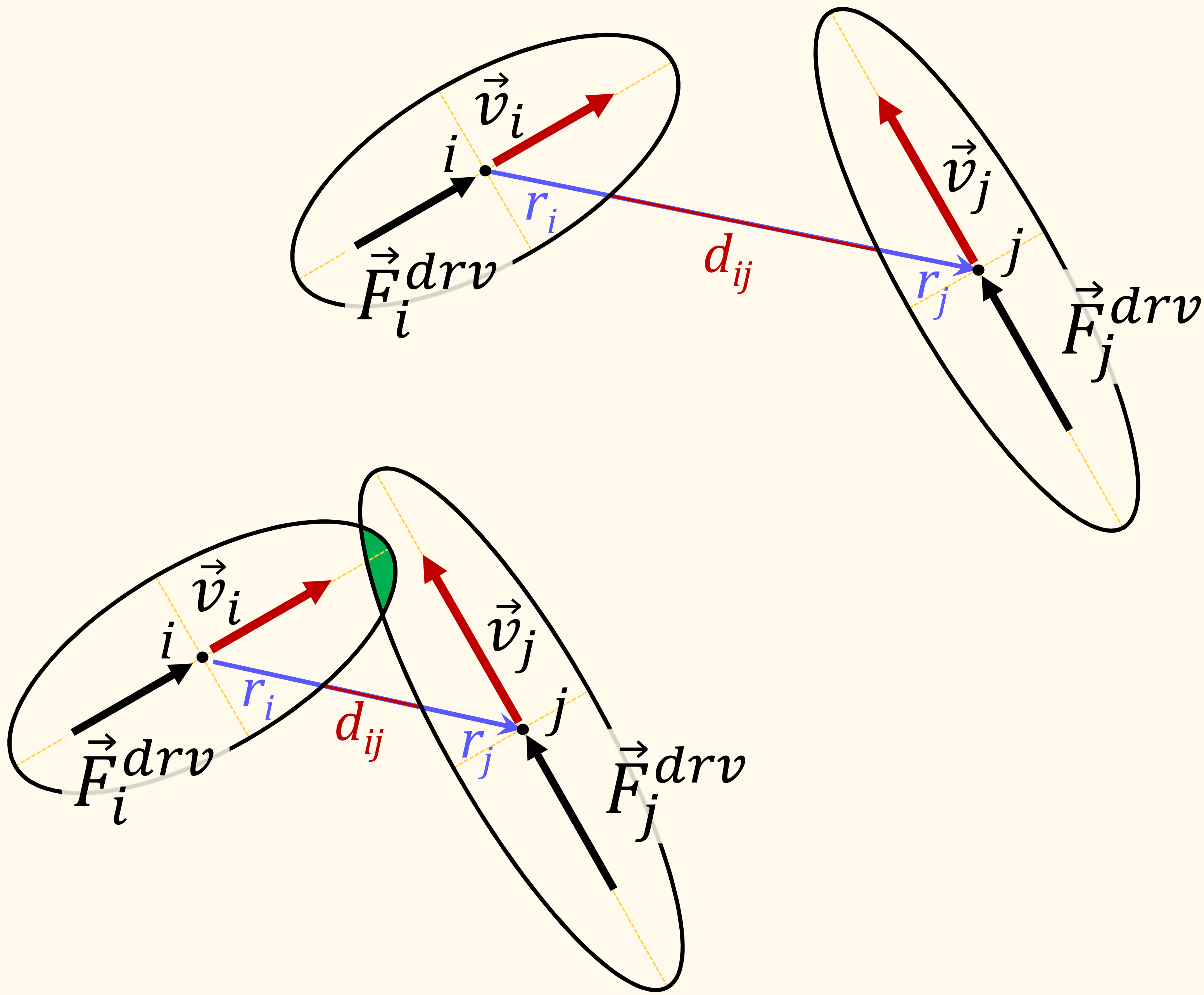


Repulsive Force:

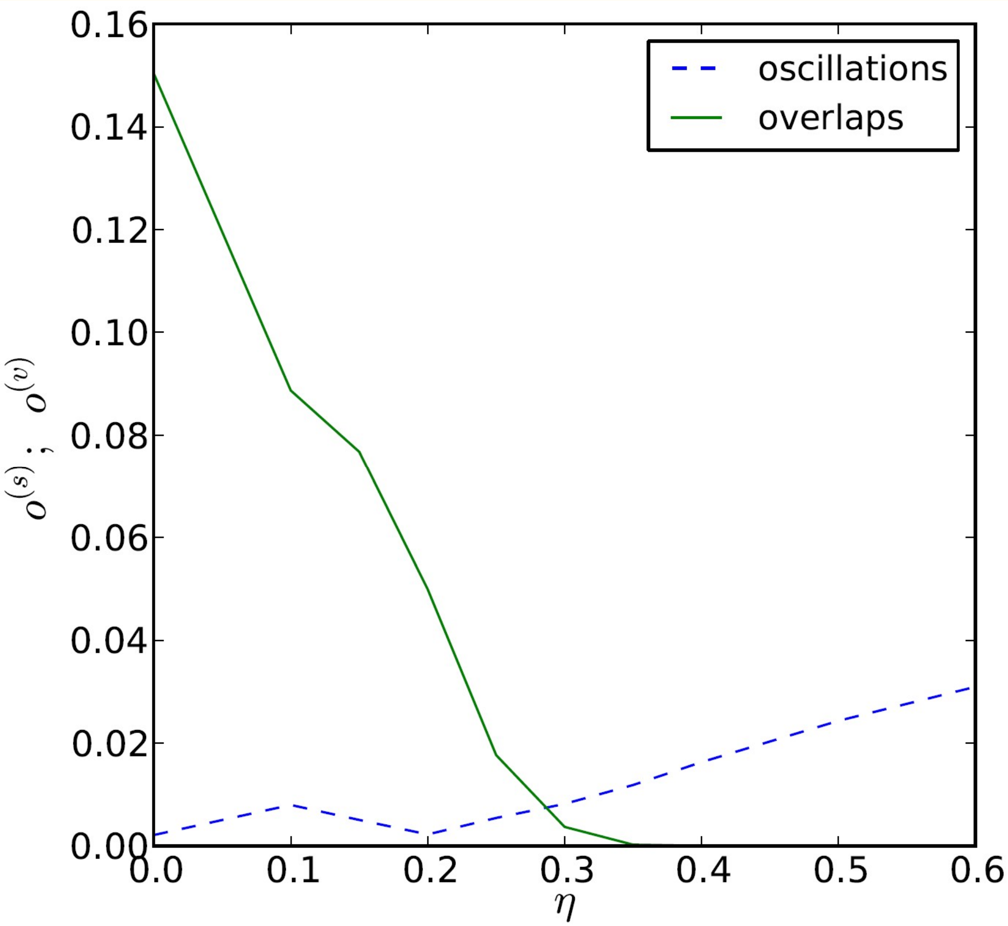
$$\vec{F}_{ij}^{rep} = -m_i k_{ij} \frac{\left(\eta \|\vec{v}_i^0\| + v_{ij}\right)^2}{d_{ij}} \frac{\left(\vec{R}_j - \vec{R}_i\right)}{\left\|\left(\vec{R}_j - \vec{R}_i\right)\right\|}$$

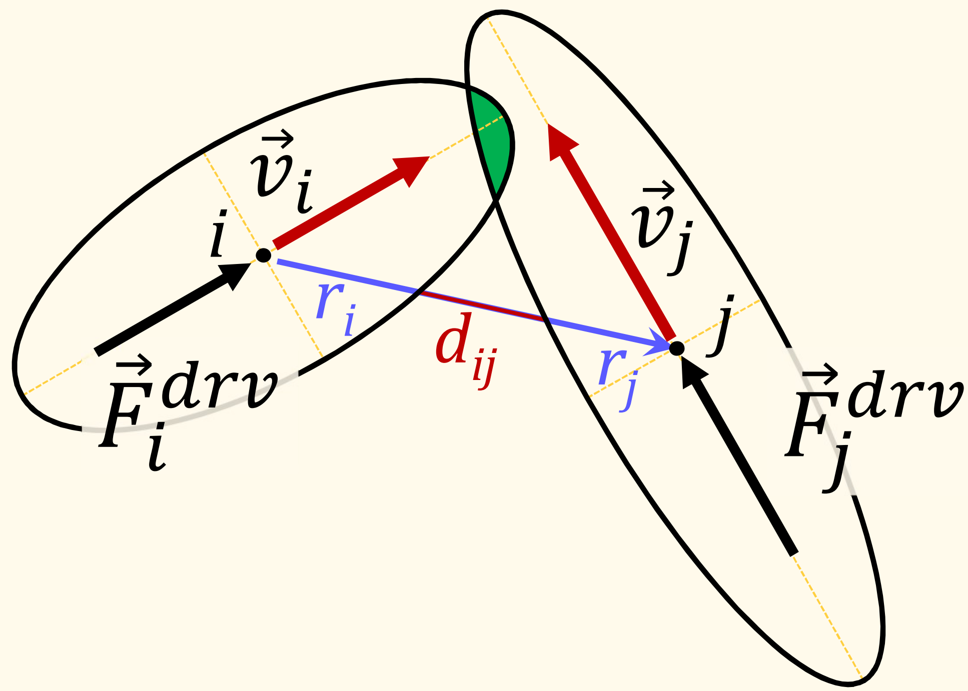
Chraibi, M., Seyfried, A. and Schadschneider, A. (2010), "Generalized centrifugal-force model for pedestrian dynamics," *Physical Review E*, **82**:4, p. 046111.

Overlap and Oscillations



Generalized Centrifugal-Force Model



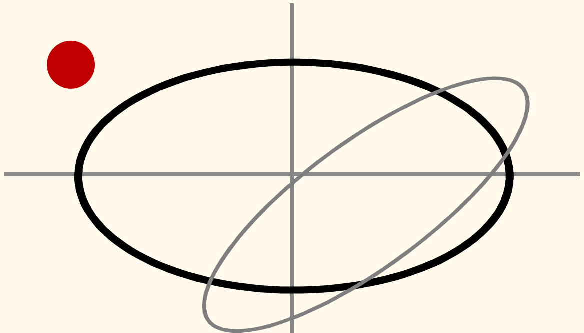


Overlapping Proportion:

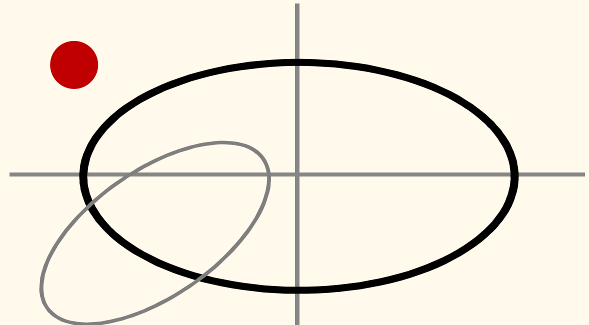
$$o^{(v)} = \frac{1}{n_{ov}} \sum_{t_0}^{t_f} \sum_{i=1}^{i=N} \sum_{j>i}^{j=N} \frac{A_{ij}}{min(A_i, A_j)}$$

Chraibi, M., Seyfried, A. and Schadschneider, A. (2010), "Generalized centrifugal-force model for pedestrian dynamics," *Physical Review E*, **82**:4, p. 046111.

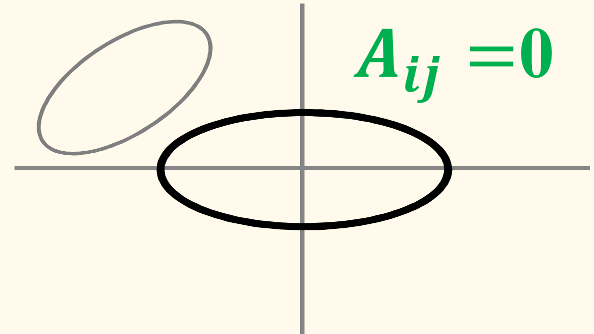
Relative Position Classification



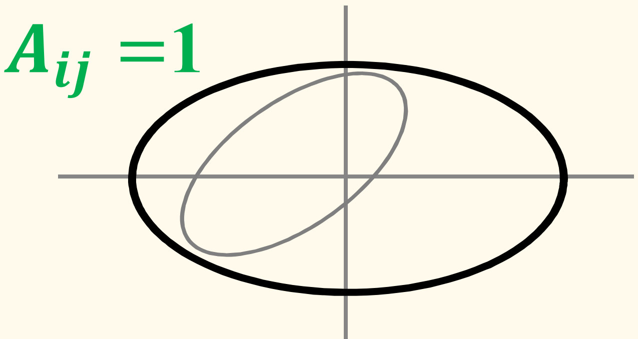
1: Transversal at 4 Points



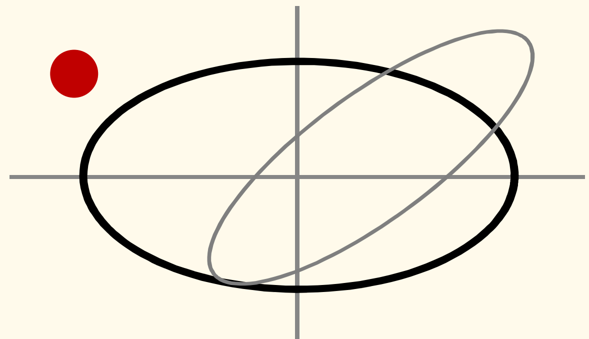
2: Transversal at 2 Points



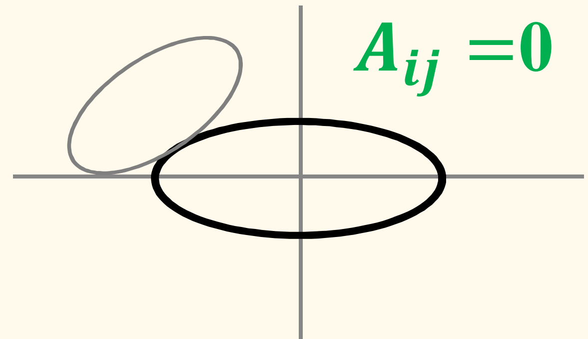
3: Separated



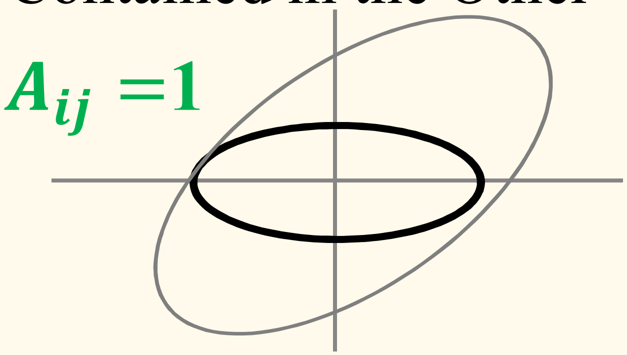
4, 5: One Ellipse Contained in the Other



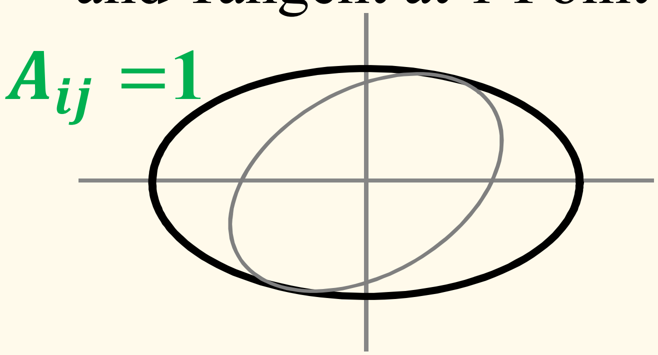
6: Transversal at 2 Points and Tangent at 1 Point



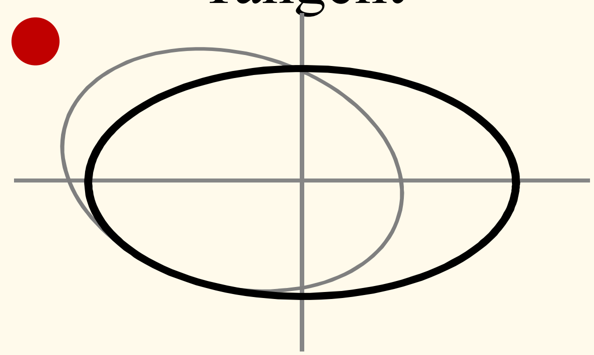
7: Externally Tangent



8: Internally Tangent at 1 Point

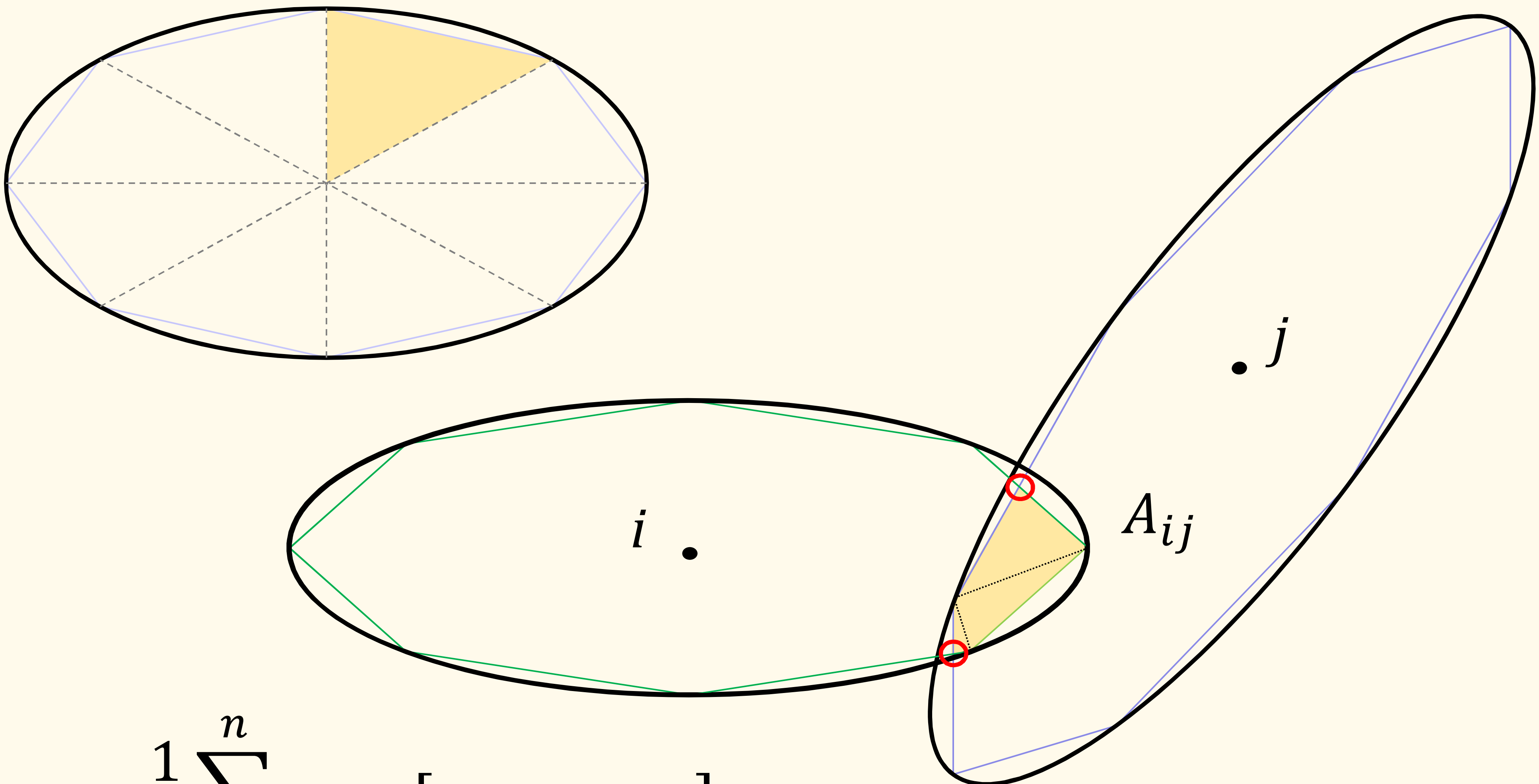


9: Internally Tangent at 2 Points



10, 11: Osculating and Hyperosculating

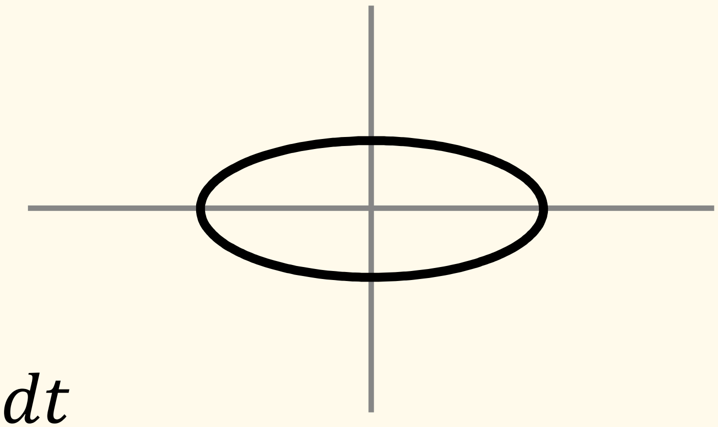
Overlap Area: Inscribed Polygons



$$A_{ij} = \frac{1}{2}\sum_{k=1}^{n} | \, x_{1,k}\left[y_{2,k} - y_{3,k}\right] + x_{2,k}\left[y_{3,k} - y_{1,k}\right] + x_{3,k}\left[y_{1,k} - y_{2,k}\right] |$$

Ellipse Area by Gauss-Green Formula

$$\left.\begin{aligned} x(t) &= A \cdot cos(t) \\ y(t) &= B \cdot sin(t) \end{aligned}\right\} \quad 0 \leq t \leq 2\pi$$



$$A = \frac{1}{2}\int_{t_1}^{t_2} [x(t) \cdot y'(t) - y(t) \cdot x'(t)]\, dt$$

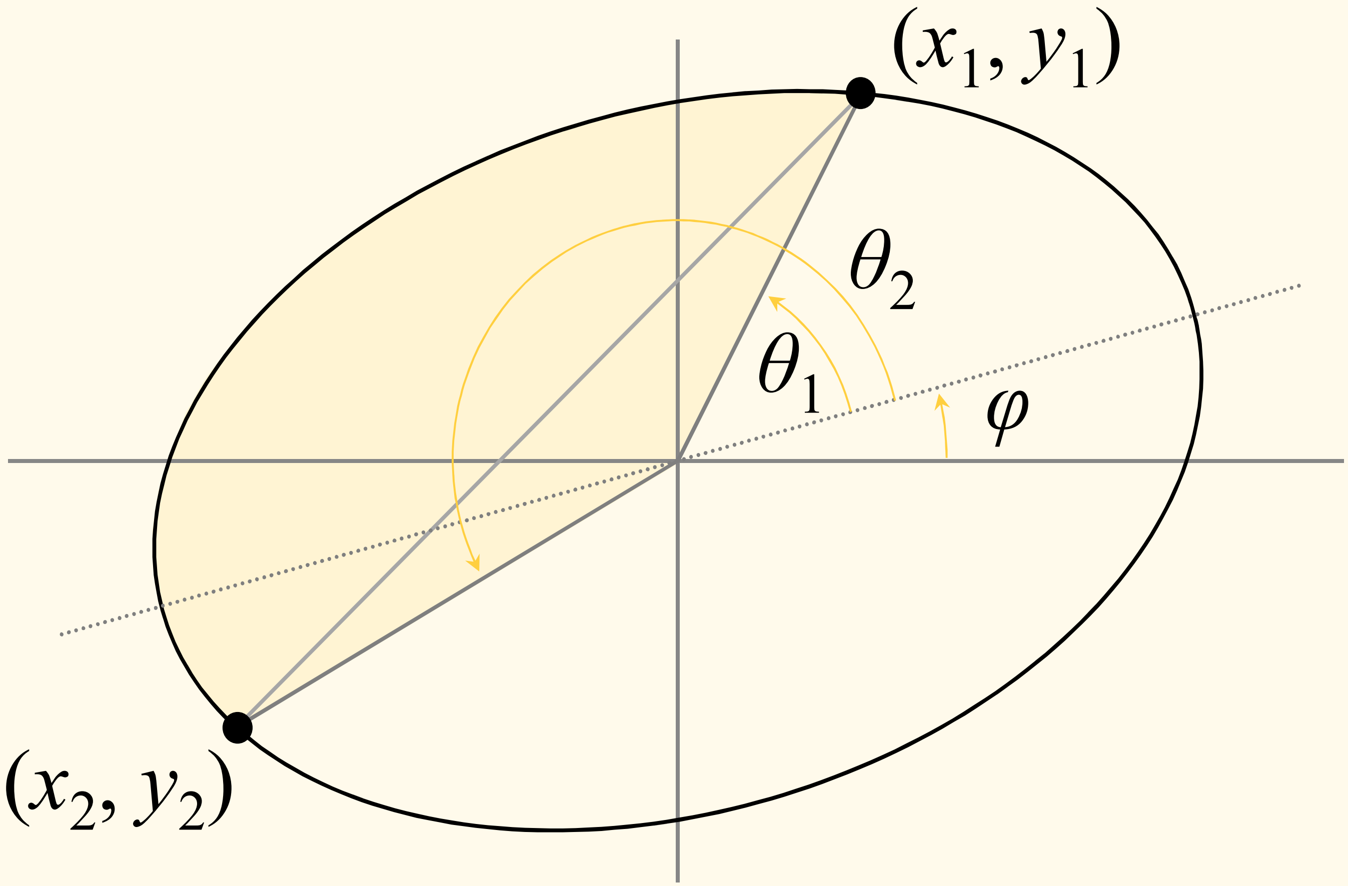
$$= \frac{1}{2}\int_{0}^{2\pi} [A \cdot \cos(t) \cdot B \cdot \cos(t) - B \cdot \sin(t) \cdot (-A) \cdot \sin(t)]dt$$

$$= \frac{1}{2}\int_{0}^{2\pi} A \cdot B \cdot \left[\cos^2(t) + \sin^2(t)\right]dt$$

$$= \frac{A \cdot B}{2}\int_{0}^{2\pi} dt$$

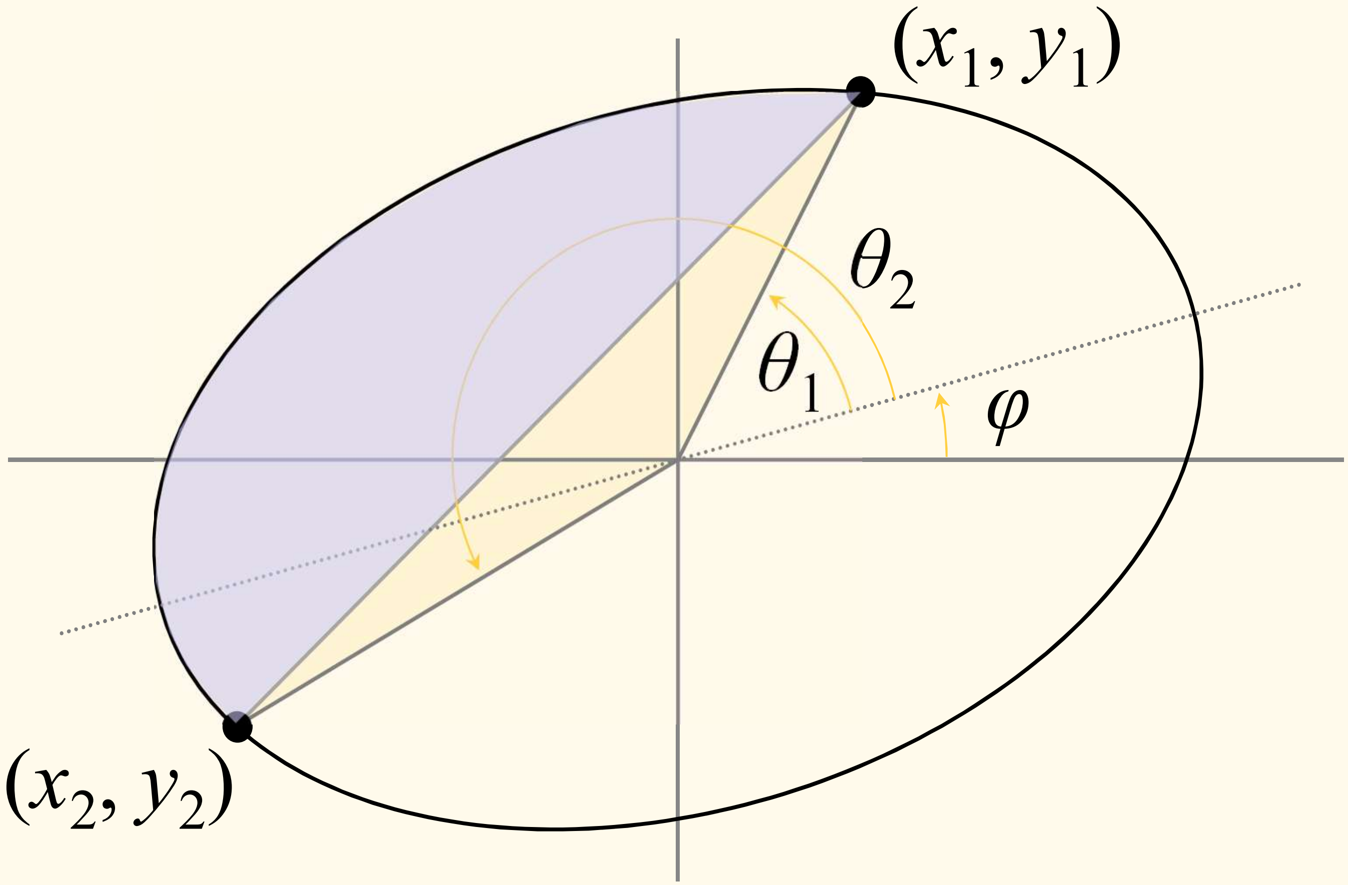
$$= \pi \cdot A \cdot B$$

Ellipse Sector Area



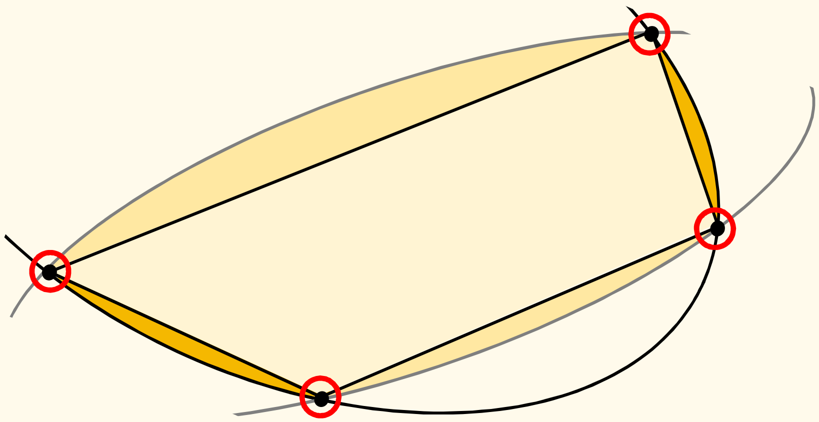
$$\text{Sector Area} = \frac{A \cdot B}{2} \int_{\theta_1}^{\theta_2} dt = \frac{(\theta_2 - \theta_1) \cdot A \cdot B}{2}$$

Ellipse Segment Area

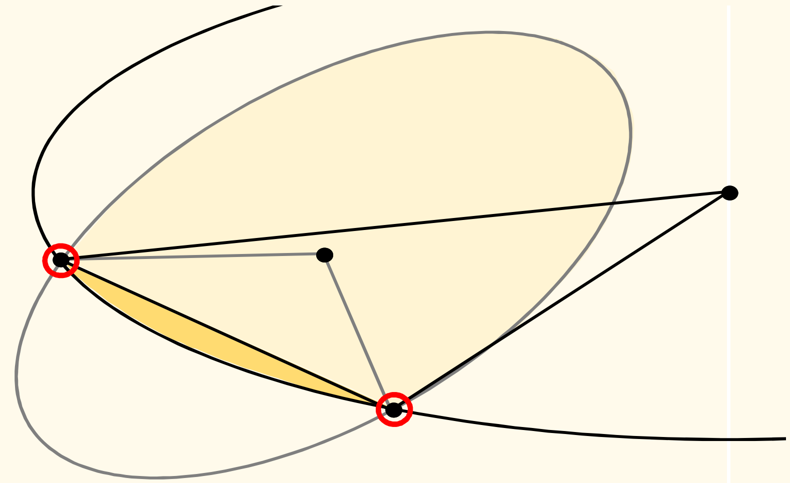


$$\text{Segment Area} = \frac{(\theta_2 - \theta_1) \cdot A \cdot B}{2} \pm \frac{1}{2} \cdot |x_1 \cdot y_2 - x_2 \cdot y_1|$$

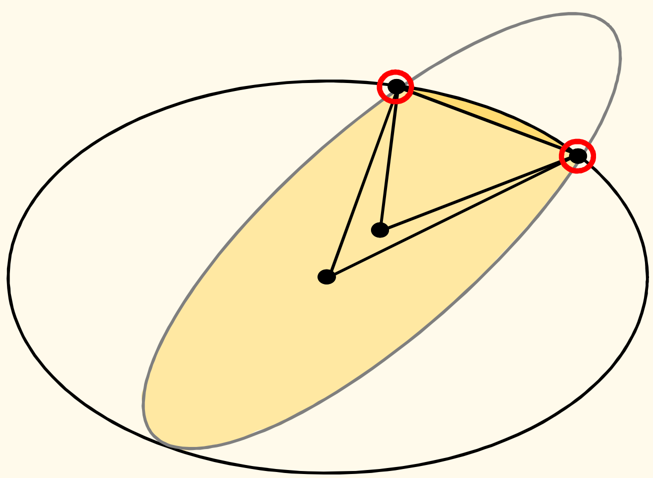
Ellipse Overlap Area



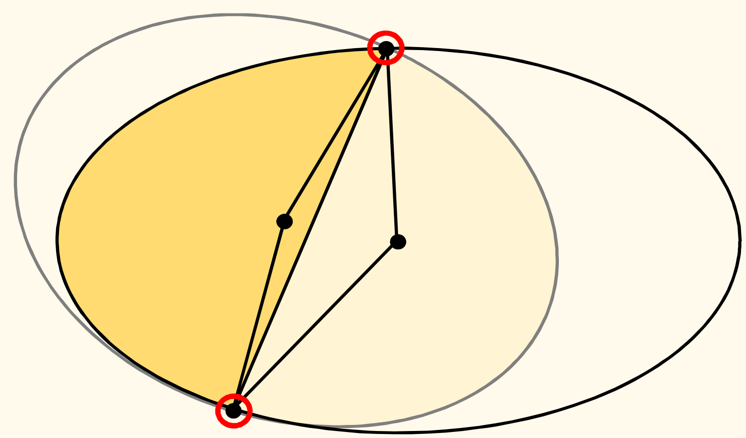
Relative Position 1



Relative Position 2



Relative Position 6



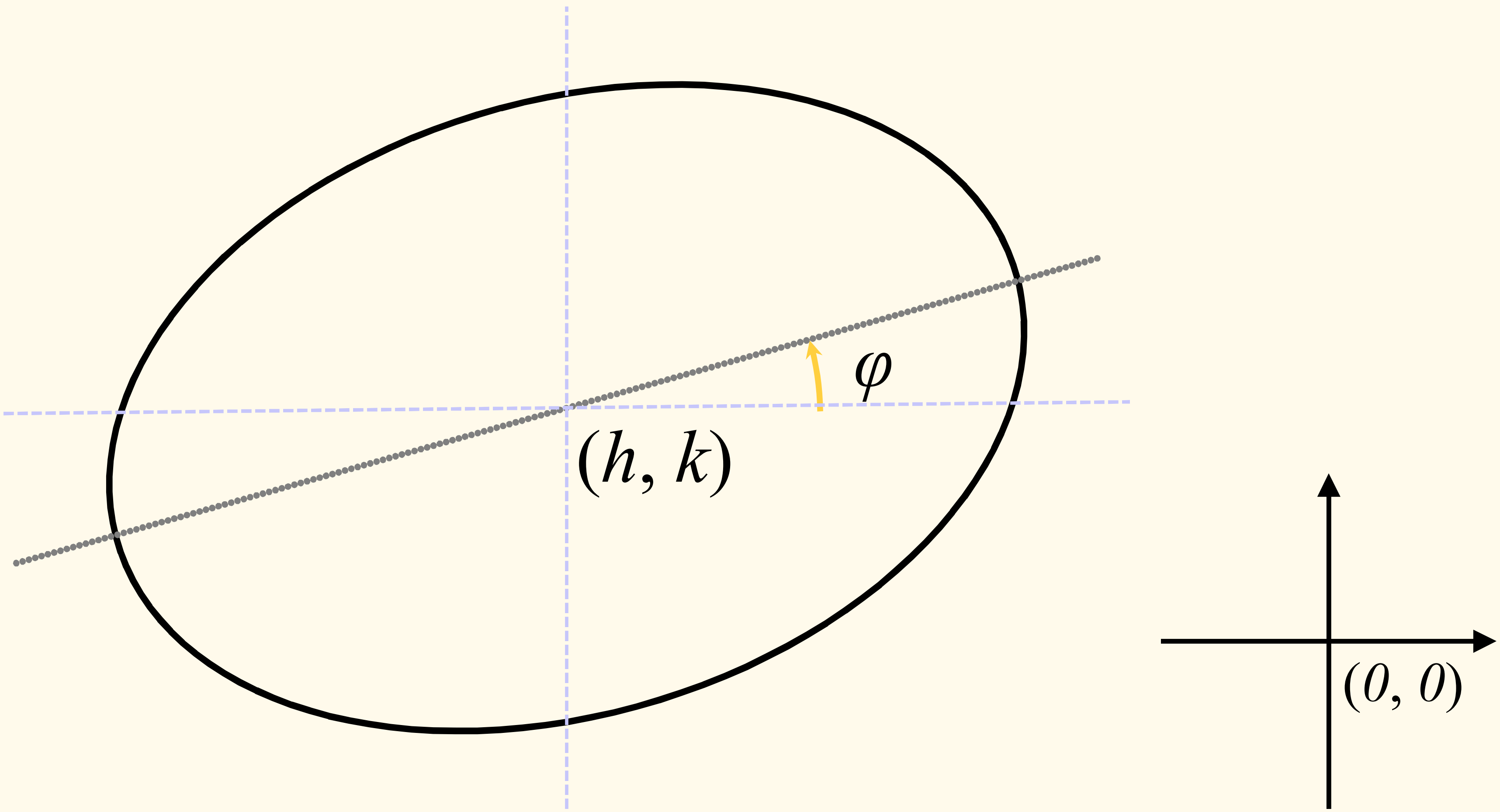
Relative Positions 10, 11

Hughes, G.B., and Chraibi, M. (2014), "Calculating Ellipse Overlap Areas," *Computing and Visualization in Science* **15**, pp. 291-301.

Intersection Points

General Ellipse (Parametric)

$$\left.\begin{aligned} x(t) &= A \cdot cos(\varphi) \cdot cos(t) - B \cdot sin(\varphi) \cdot sin(t) + h \\ y(t) &= A \cdot sin(\varphi) \cdot cos(t) + B \cdot cos(\varphi) \cdot sin(t) + k \end{aligned}\right\} \quad 0 \le t \le 2\pi$$



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Intersection Points

General Ellipse (Implicit Polynomial)

$$AA \cdot x^2 + BB \cdot x \cdot y + CC \cdot y^2 + DD \cdot x + EE \cdot y + FF = 0$$

$$AA = \frac{\cos^2(\varphi)}{A^2} + \frac{\sin^2(\varphi)}{B^2}$$

$$BB = \frac{2 \cdot \sin(\varphi) \cdot \cos(\varphi)}{A^2} - \frac{2 \cdot \sin(\varphi) \cdot \cos(\varphi)}{B^2}$$

$$CC = \frac{\sin^2(\varphi)}{A^2} + \frac{\cos^2(\varphi)}{B^2}$$

$$DD = \frac{-2 \cdot \cos(\varphi) \cdot [h \cdot \cos(\varphi) + k \cdot \sin(\varphi)]}{A^2} + \frac{2 \cdot \sin(\varphi) \cdot [k \cdot \cos(\varphi) - h \cdot \sin(\varphi)]}{B^2}$$

$$EE = \frac{-2 \cdot \sin(\varphi) \cdot [h \cdot \cos(\varphi) + k \cdot \sin(\varphi)]}{A^2} + \frac{2 \cdot \cos(\varphi) \cdot [h \cdot \sin(\varphi) - k \cdot \cos(\varphi)]}{B^2}$$

$$FF = \frac{[h \cdot \cos(\varphi) + k \cdot \sin(\varphi)]^2}{A^2} + \frac{[h \cdot \sin(\varphi) - k \cdot \cos(\varphi)]^2}{B^2} - 1$$

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Intersection Points

$$u_2 \cdot x^2 + u_1 \cdot x + u_0 = 0$$

$$u_2 = (AA_1), \qquad u_1 = (BB_1 \cdot y + DD_1), \qquad u_0 = \left(CC_1 \cdot y^2 + EE_1 \cdot y + FF_1\right)$$

$$v_2 \cdot x^2 + v_1 \cdot x + v_0 = 0$$

$$v_2 = (AA_2), \qquad v_1 = (BB_2 \cdot y + DD_2), \qquad v_0 = \left(CC_2 \cdot y^2 + EE_2 \cdot y + FF_2\right)$$

Bézout determinant:

$$(u_1 \cdot v_0 - u_0 \cdot v_1) \cdot (u_2 \cdot v_1 - u_1 \cdot v_2) - (u_2 \cdot v_0 - u_0 \cdot v_2)^2 = 0$$

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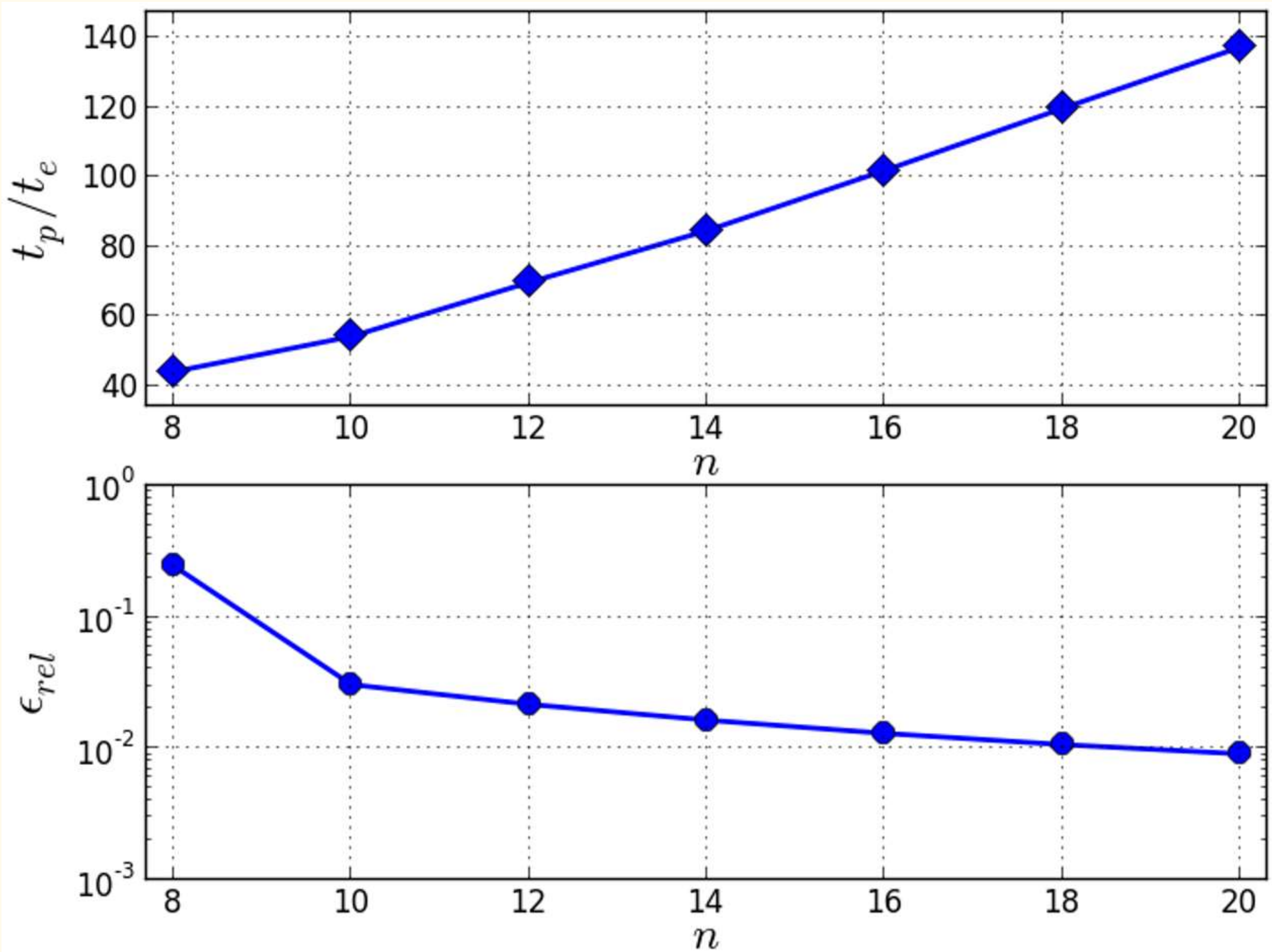
Run-Time Comparison



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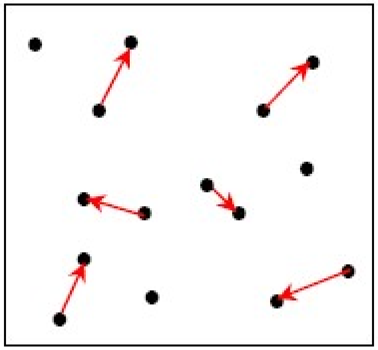
Validation: Spatial Randomness

List of discrete points in a continuous 2D domain:
$\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$

Each point has a nearest neighbor at a specific distance:
$\{D_1, D_2, \ldots, D_n\}$



Random Sample of $m \approx 0.1$ n Nearest-Neighbor Distances

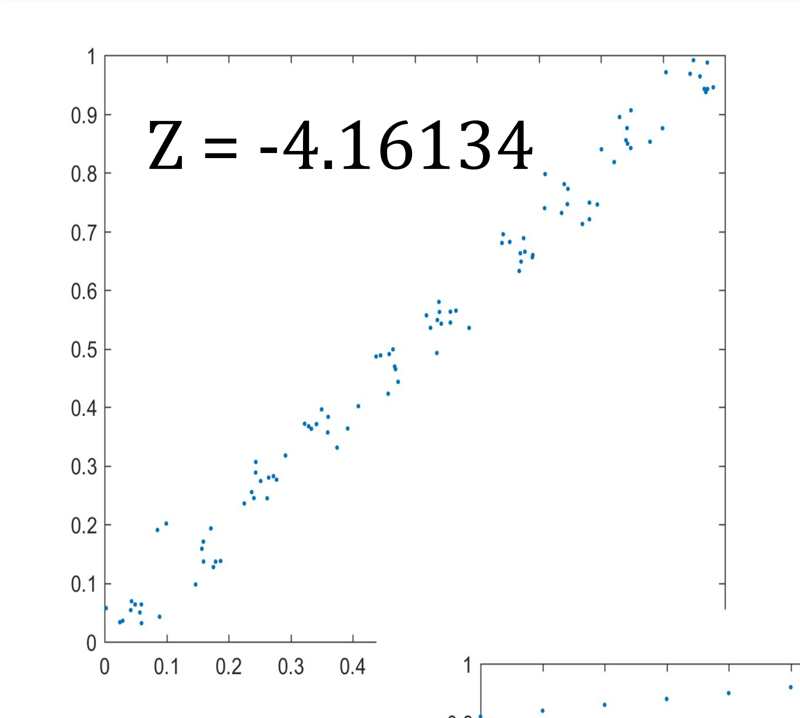
λ = point density within the domain
= n / area

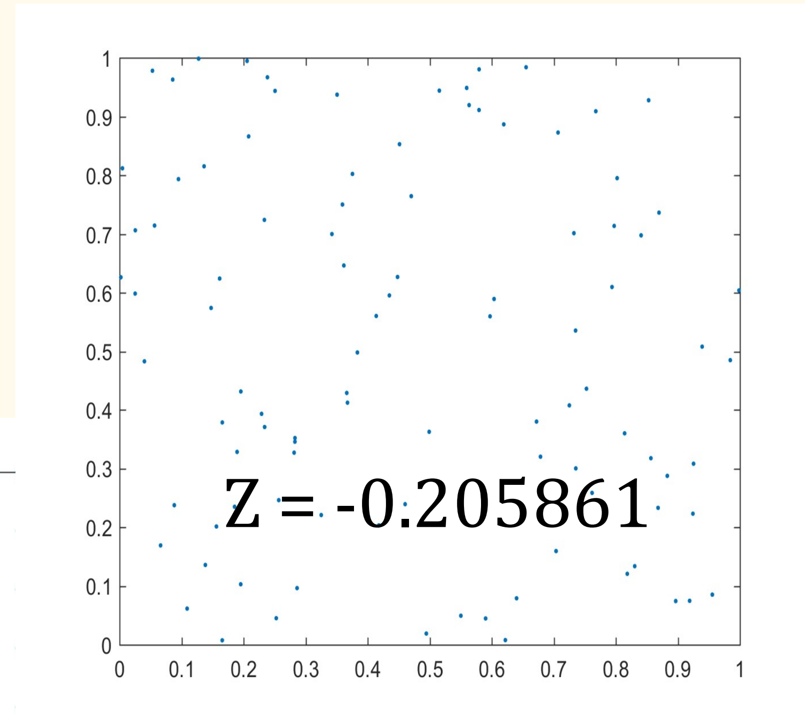
Test Statistic with Standard Normal Distribution:

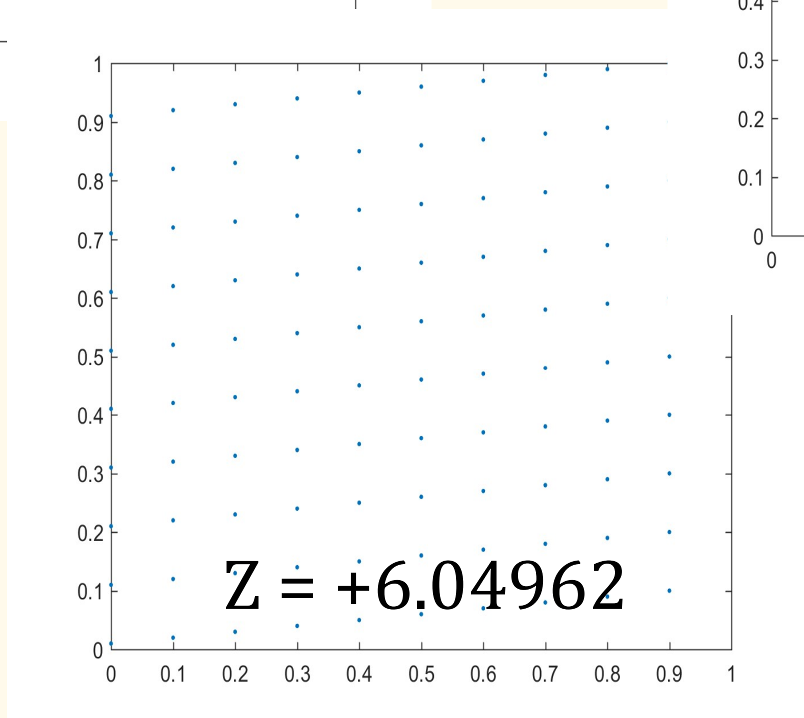
$$Z = \frac{\bar{d}_m - \frac{1}{2\sqrt{\lambda}}}{\sqrt{\frac{4-\pi}{4\pi m \lambda}}} \sim N(0,1)$$

Clark-Evans Test Statistic

n = 100 discrete points
within [0, 1]x[0, 1]





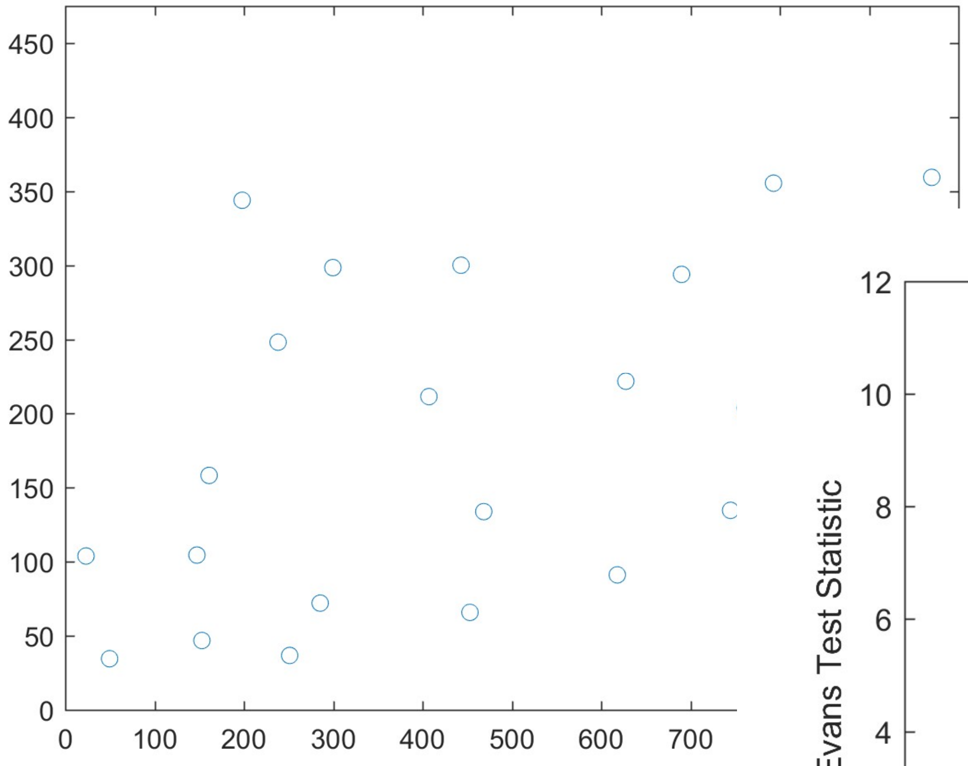


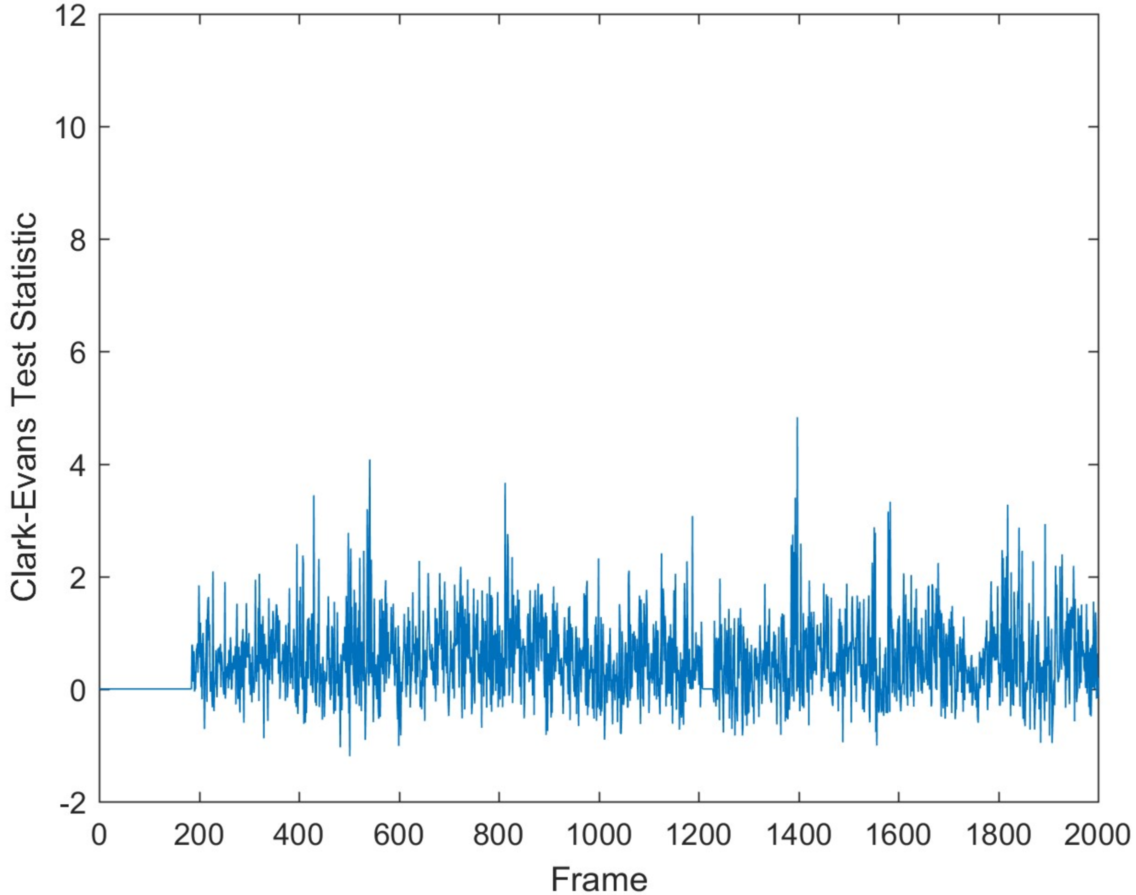
Deviations from a "2D Poisson Process":

More Regular (Z > 0)

More Clustered (Z < 0)

Spatial Randomness of Pedestrian Flow





- Corridor, bidirectional flow
- http://ped.fz-juelich.de/experiments/2013.06.19_Duesseldorf_Messe_BaSiGo/result/corrected/BI_CORR.zip
- bi_corr_400_b_02.txt

Questions