SOCIAL AND PHYSICAL PEDESTRIAN SIZES AND THEIR IMPACT ON THE DECISION-BASED MODELING

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ABSTRACT

During the pedestrian movement in an environment, every attendant contributes to the total crowd mass by their own (physical) volume. However, this is not appropriate enough to describe pedestrian behavior which is an important aspect to build or improve a pedestrian model and make it predictable. This paper deals with the model features considering a social impact on pedestrian size. Even though the physical (minimum) pedestrian size can be known and estimated from the real data using distance of human shoulders, i.e. is fixed, it tells us nothing about their current (social) size. Moreover, the fact that every pedestrian has the same physical size does not imply that they have the same social size in the current time - the social size can vary during the time with respect to the surroundings of the pedestrian. Furthermore, the sizes how the pedestrian are seen and how they feel are different, i.e. pedestrians decide about their own (social) compression in a crowd. To understand these properties properly, the author's decision-based model is introduced including ruling principles capturing pedestrian movement in both free flow and congestion. The impact of social and physical sizes on prediction power of the model is discussed.

INTRODUCTION

In the last few decades, the modeling of pedestrian dynamics went into the agent-based scientific stage which is possible due to the better computational performance (Schadschneider et al, 2018). This new research direction cleared the way for a lot of possible model improvements - one of them is a concept of the agent size which may be variable in time, according to circumstances in the agent surroundings. This idea is originally based on the proxemics theory – the invisible bubble around the body is called proxemics and its size depends on both the biological processes and the cultural environment of the individual (Hall, 1959; 1966). However, we cannot confuse social distance in the agent-based modeling (Was, 2010) which differs from current social distancing as the prevention of the pandemic situation (Adams, 2020). In this contribution, we understand the pedestrian size as the invisible bubble with size coming from the agent properties and not ordered by any state government.

The models can be classified according to a lot of criteria (Cheng et al, 2014; Schadschneider et al, 2018). For a great promise for the future, we choose to develop new continuous (in both time and space) microscopic model to maintain the real human behavior pattern using the microscopic point of view due to the possibility to implement the individual phenomena. The specific model type is chosen according to the task we want to solve (Seyfried et al, 2006). Developing of pedestrian models with simple and realistic principles (Kang et al, 2017) or capturing rational behavior using perception and decision stages of pedestrian movement (Bailo et al, 2018) are examples of the contemporary topics. To capture the agent moving process properly (Bailo et al, 2018; Moussaid et al, 2011), we choose to let the pedestrians make their own decisions in accordance with conditions in their neighborhood. Therefore, we build continuous microscopic decision-based model. There are many

benefits of this choice – since heterogeneity in pedestrian behavior is observable (Bukáček et al, 2018; Chattaraj, 2009; Schadschneider, et al 2010), the decision-based pedestrian model can be simply generalized to predict the movement of many different groups of people (Lovreglio et al, 2016). The choice behavior during evacuation process in a building (Zhu, Shi, 2016) and pedestrian competitiveness (Garcimartin et al, 2016) can be also considered.

During pre-calibration studies, an agent size was shown as one of the crucial features of our decision-based model significantly affecting the output flow from a simulation room. Before the calibration itself, it is necessary to study the behavior in pedestrian surroundings to examine how they change their social size (in the sense of proxemics) and if it fits our initial suggestions. For this purpose, we choose the density as the statistic describing the surroundings properly. However, we need a specific individual approach to estimate the density in a microscopic way. There is lots of ways to compute the density values coming from the physical definition. Many authors dealt with a unification of existing processes of evaluating the fundamental quantities (including flow and velocity) during the data analysis (Seyfried et al, 2009; Steffen, Seyfried, 2010; Tordeux, 2015). This unified concept brings a new possibility to take the microscopic aspect for our computation into consideration. Therefore, we use an approach using pedestrian distribution to estimate the individual pedestrian density in their surroundings of a specific shape.

Thus, this contribution presents the author's decision-based pedestrian model, its rules and strong sides including social and physical pedestrian size, which is followed by calibration strategy. Then the time and spatial development of the pedestrian social size are presented using the individual density concept promoted above. Finally, the impact of the social and physical sizes on prediction power of the model is discussed.

DECISON-BASED PEDESTRIAN MODEL

Our model is defined in detail in (Vacková, Bukáček, 2019), therefore just a brief overview follows. It is necessary to mention that every rule of pedestrian movement can be scaled using corresponding parameters, the list of significant of them is in Table 1. The design of the experiment is also adjustable.

Model Definition

In general, pedestrian movement is comprised of several phases - strategic, tactical and operational (Cheng et al, 2014; Kang, Han, 2017; Martinez-Gil et al, 2017). A strategic phase defines the global (long-term) plan of the pedestrian movement. As the strategy is chosen, the pedestrian needs to find their course of action, i.e. the pedestrian searches locally for possible paths (local targets) - a tactical phase (short-term decisions). And, finally, the specific next position taken by the pedestrian to reach their chosen goal from strategic phase represents an operational phase. The strategic phase is evident due to the design of our experimental data used for future calibration (Bukáček et al, 2018) (passage of pedestrians through a room right to the exit). Thus, the following text treats tactical and operational phases.

Tactical Phase

Tactical phase represents the selection of local targets to move the pedestrian straight to their goal. The goal is, in our case, designated as reaching the exit in the room. Although it can be made a dense grid of local points (Pathfinder User Manual, 2019), we bring the concept of (few) checkpoints (local targets) which can be widened into a grid as well. The need of a grid is replaced by sorting the checkpoints into the different levels. The experiment room geometry (Bukáček et al, 2018) without any obstacle does not involve any other checkpoint than in the exit area. Although one checkpoint there would be enough regarding the concept described above, there are several of them to smooth the pedestrian leaving process - any point in the exit door width is considered as the checkpoint.

Operational Phase

At this stage, our pedestrian has their global aim and sees their tactics few steps forward, thus the one final thing remains - quantification of their new coordinates for the next time step. During this procedure they have to avoid any inadmissible places, namely walls, potential obstacles and mainly other pedestrians. We built a set of rules which brings us a non-collision system.

Blind Velocity

Every pedestrian has an optimum (i.e. desired) speed for their movement. Firstly, the pedestrian tries to find their new position using the optimum (i.e. blind) direction in Figure 1, which is calculated as the one to the current checkpoint, and they keep moving with the desired speed or accelerate.

Figure 1: Operational phase: the blind velocity. They accelerate or keep moving with the desired speed.

Collision Avoidance

If the blind velocity fails, i.e. the optimum position is not available, the pedestrian needs to change the optimum direction due to the possible collision with the room equipment or the other pedestrians. As we still want the pedestrian to profit as much as it is possible, the change of the course is minimized if there are more possibilities and changing their course is examined primarily (it does not change the pedestrian speed which can be still increased) - Figure 2. If the change in the pedestrian course does not solve the conflict, the slowing down is admitted in Figure 3. If there is more than one admissible position after the pedestrian slows down, the deceleration is minimized. Let us mention that the combination of slowing down and course change is not included yet - with respect to the considered pedestrian step size, it should not play an important role or cause a significant difference.

Figure 2: Operational phase: collision avoidance by course change - the blind velocity, searching for admissible positions, the final position.

Figure 3: Operational phase: collision avoidance by slowing down - the course change does not work, searching for positions by shortening the blind distance, the final position.

Dense Crowd Behavior

According to our rules mentioned up to now, we can observe clogging - the formation of self-stabilizing arch (bridge) (Garcimartin et al, 2014, Schadschneider et al, 2018). It is plentifully studied in granular systems (Hidalgo et al, 2013) and does not last indefinitely in reality either, thus we need to guarantee that it will vanish on its own also in our model behavior. Hence another set of microscopic rules is needed.

We assume that all pedestrians have the same *initial size* $s > 0$ when they come into the system and, as they move towards the exit in time $t > 0$, we allow to change their size in order to cover the crowd behavior when the exit capacity is almost reached – we call it pedestrian *social size* $s_\alpha(t)$. Their social sizes are reduced only to themselves - they see each other still at the initial size s and, only from their perspective, they may decide about their own pressing, i.e. social compression. This behavior reflects the real ability of pedestrian to pass through narrow space using body rotation or pressing. The minimum possible pedestrian (social) size is called pedestrian *physical size* $\tau_s > 0$. Initial and physical sizes are model parameters (see Table 1) and it is fulfilled that $0 < \tau_s \leq s_\alpha(t) \leq s$. The physical size can be estimated from the real data using distance of human shoulders. However, it is possible to design the physical size smaller than the real one in the model if it is required for the modeling purpose. The pedestrian social size is nonincreasing function, i.e. the pedestrian is not allowed to expand again if their path is free again – the function trend considers our experimental data used for model calibration with evacuation of one room through one exit. For this kind of experiment, it is sufficient this specific resizing; for different experimental settings, the model can be easily upgradable for an arbitrary trend in the function of pedestrian social size.

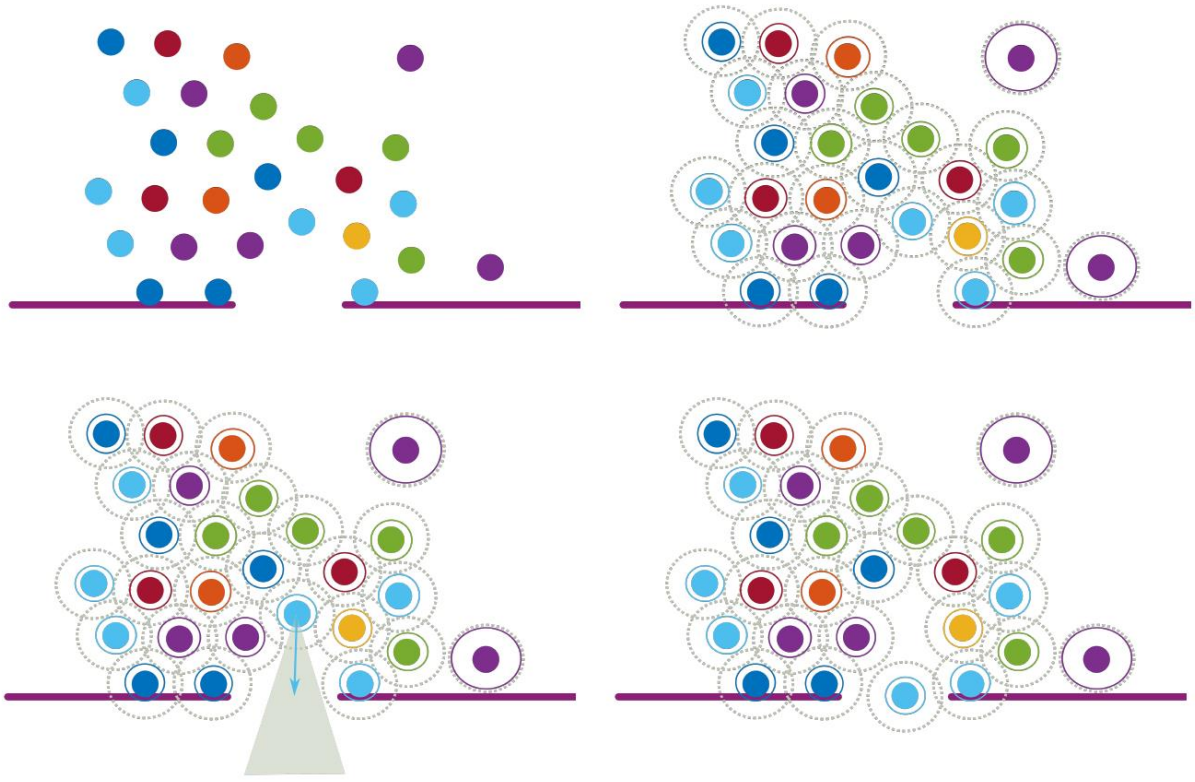

Figure 4: Operational phase: the dense crowd behavior - when an arch occurs in the exit area and the solution by crisis model rules is provided. Solid circles around pedestrians represent the pedestrian social size which changes in time, dotted circles depict the initial size.

This resizing can make the flow from the room smooth again. When the reducing of pedestrian (social) sizes is not sufficient for non-zero bottleneck flow, i.e. the exit area is stuck and the pedestrian is the part of the arose arch, a crisis rule is then applied – the pedestrian looks in a specific angle and if there is a free space, the pedestrian leaves the arch and goes right to the exit from the room, see Figure 4. Otherwise, the pedestrian has no place to go due to the crowd and their speed becomes zero.

Model Possible Improvements

Although every pedestrian has the same initial value of parameters at the current stage of our research, the model is easily upgradable to one with heterogeneous population or complex structures.

It is possible to put different kind of people into a simulation. We may define some pedestrians as family members, i.e. parents and children which are connected to move together as a one small compact group, then parents could have a special skill to change locally their optimum direction to the exit to find their lost children etc.; or with different age using empirical age distribution; or as differently disabled persons, for example person in wheelchair, blind person with their assistance dog etc.; or as confused people, for instance during the evacuation from a smoky room, by randomization tactical decision phase; or as people with different aggressiveness, i.e. skill of winning conflicts. We may define by appropriate checkpoint series different pedestrian strategies, for example a security guy walking around the building every hour (start and finish are the same points); or students at the end of break at high school running into their classes at the same time (different finish points); or waiter and waitress (with lots of stops and regular target).

It is possible to define a complex room structure. We may implement the obstacles of arbitrary shape; or stairs, i.e. an area with a specific coefficient for changing pedestrian speed; or T-junction and intersection, i.e. area with crossing flows etc.

CALIBRATION STRATEGY

We have the model with defined rules and principles using parameters summarized in Table 1 which are set up by physical laws. This setting does not have to be the one which produces a real system. The aim of the calibration process is to find the eligible parameter values. Nevertheless, there is not a single correct way to calibrate the model - there is no universally right method (Campanella, 2014). Different methods are used according to the type of the model, its following application and, of course, the author's preferences. The choice of the quantity, which represents the data properties, and the objective function, which describes the error between model and real data, is very difficult.

Table 1: List of significant model parameters and their explanations.

Parameter Mark	Unit	Meaning
w	m	Minimum distance from a wall etc.
s	m	Initial size of a pedestrian
δs	m	Step to reduce the pedestrian size
τ_s	m	Minimum (physical) pedestrian size
v_{opt}	m/s	Pedestrian optimum speed
ν	rad	Pedestrian field of vision
φ	rad	Maximum change of a pedestrian course
a	$\mathrm{m/s^2}$	Pedestrian acceleration
a_{crisis}	$\mathrm{m/s^2}$	Acceleration if an arch occurs
ϑ	rad	Field of vision if an arch occurs
n_{CP}	-	Current checkpoint benefit

Our calibration concept consists of calibration episodes. We want to avoid choosing just a few metrics and quantities to describe the whole, complex system and rising problems with finding the global optimum. Thus, we design the calibration episodes which are separate and every of them covers one type of pedestrian behavior captured by (one or several) model parameters.

We need to start at the very beginning of the pedestrian movement, firstly capture the basic properties of the movement process, secondly examine more interesting behavior like follower-leader (Vacková, Bukáček, 2019) etc. Only this course of action during the calibration assures that our model will fit the real system with its qualitative phenomena. Due to the well-defined model rules in the previous text, we know it is possible to set the parameters to obtain a non-stuck system - we made a detailed discussion in (Vacková, Bukáček, 2019).

As a very first step before calibrating parameters considering pedestrian size, we decided to study how they influence both individual and group pedestrian behavior in the simulation room. We performed simulations for 336 parametric sets, each of them in 10 iterations and with inflow 1.5 ped/s and experimental time 120 s – we assume $s \in \{0.1, 0.11, \dots, 0.3\}$, $\tau_s \in \{0.05, 0.06, \dots, 0.3\}$ and $\tau_s \leq s$ (the step of resizing is set to $\delta s = 0.05$ m; to have almost continuous social size function it has to be chosen much smaller).

MICROSCOPIC INSIGHT INTO PEDESTRIAN SIZE

In the microscopic way, our idea is to compare pedestrian social size $s_\alpha(t)$ with individual statistics describing the pedestrian surroundings and then examine if their trends correspond to our expectation or not.

Extraction of Individual Quantities

There is not the only way how to estimate the quantities, as was discussed above. Therefore, the estimates will be defined at first.

Individual Density

We assume every single pedestrian to be a source of (individual) density distribution. Let us rewrite the definition of density in an area A as an estimate using kernel distribution theory (Baszczynska et al, 2016; Chacon, Duong, 2018; Wand, Jones, 1994).

$$\rho_A = \frac{N}{|A|} = \frac{\int_A p(\vec{x})\, d\vec{x}}{|A|} = \frac{\int_A \sum_{\alpha=1}^{N} p_\alpha(\vec{x})\; d\vec{x}}{|A|} = \sum_{\alpha=1}^{N} \frac{\int_A p_\alpha(\vec{x})\, d\vec{x}}{|A|}, \tag{1}$$

where N represents the number of pedestrian, $|A|$ the size of considered area A, $p_\alpha(\vec{x})$ the individual (density) distribution generated by each pedestrian $\alpha \in \{1, 2, \dots, N\}$ and $p(\vec{x}) = \sum_{\alpha=1}^{N} p_\alpha(\vec{x})$ the density distribution in the area A. The area A does not have to be static (a detector approach) like in (Johansson et al, 2008), if it represents the surroundings of pedestrian, which can be variable in time (a dynamic approach), we call the density individual (Bukáček, Vacková, 2017; Vacková, Bukáček, 2019).

Due to the generality of (1), we can set two assumptions - the shape of the pedestrian surroundings (i.e. the area where the density is evaluated) ω_α and the shape of the individual distribution (i.e. the way how the pedestrian contributes to the total distribution). If the surroundings is set to the whole considered area and Dirac function is used as the individual distribution, the standard approach is obtained.

It is possible to use kernel defined by Voronoi cells (Seyfried et al, 2009), Gaussian distribution to blur the pedestrian smoothly to the whole area (Steffen, Seyfried, 2010), Borsalino kernel (Krbálek,

Krbálková, 2018) in a one-dimensional case, or, as we did in (Bukáček, Vacková, 2017), the conic distribution (an example is depicted in Figure 5) - its most valuable features are a bounded support, an explicit definition without the need of any numerical calculations and convergence to the Dirac function with infinitesimal radius of the base. Any shape of surroundings can be chosen as well - we choose a circular one (Bukáček, Vacková, 2017) that can be enhanced in future, for example by knowledge about body rotation (Yamamoto et al, 2019) or using a concept of visual control (Rio et al, 2014).

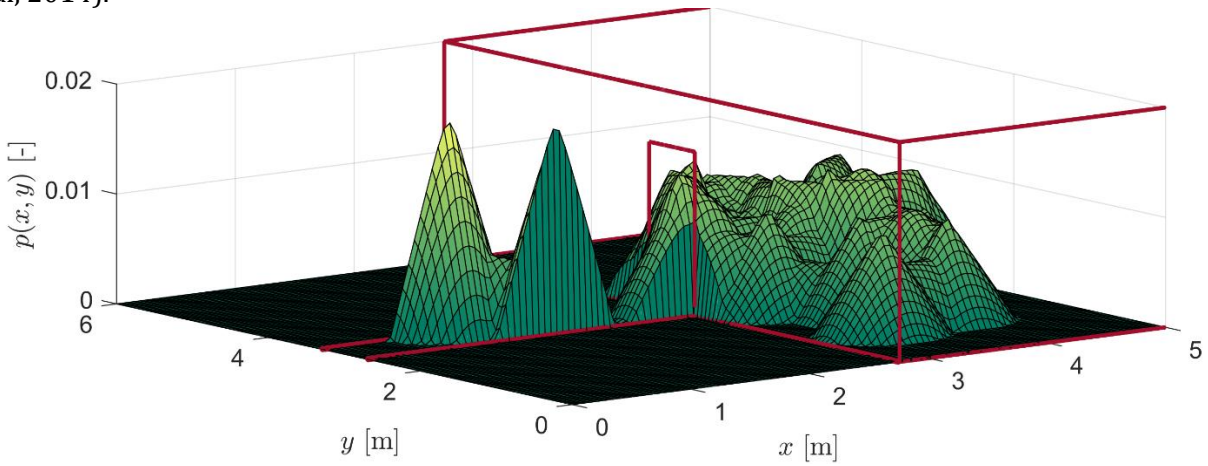

Figure 5: Density distribution with the conic individual distribution (Vacková, Bukáček, 2019).

Minimum Pedestrian Size

To obtain knowledge about the strength of compression in the simulation, we will use mean (over pedestrians) minimum (over time for each pedestrian) pedestrian size, i.e.

$$\overline{\min} s = \frac{1}{N} \sum_{\alpha} \min_{t \geq 0} s_{\alpha}(t). \tag{2}$$

Statistic and Model Speed

To compute (individual) pedestrian speed $v_{\alpha}(t)$ as statistics, we use central differences of space coordinates with moving averages to smooth pedestrian head movements. However, this kind of computation eliminates the time points when the pedestrians slow down due to the defined model rules, i.e. there is an obstacle or another agent in front of them and no possible direction to avoid them at the same time. In this analysis, for demonstration of well-defined model rules including pedestrian resizing, we need to detect these points. Thus, we take into consideration also pedestrian speed which is usually hidden in the model.

<u>Time Development of Pedestrian Social Size and Their Individual Density</u>

Individual quantities defined above are seen in Figure 6 for one chosen pedestrian α from the simulation with parametric setting $s = 0.3$ m and $\tau_s = 0.05$ m (pedestrians from other parametric settings have similar tendencies, see the next part).

Let us formulate main conclusions from Figure 6 (one subfigure after another). We can see that

- median of the individual density $\rho_{\omega_{\alpha}}$ increased with decreasing pedestrian social size s_{α};
- median of pedestrian (model) speed decreased significantly right after the first reduction of the initial size s which makes perfect sense, because the pedestrian has to slow down due to the crowd in front of them, their speed goes down to zero and then they reduce their size (according to the model rules described in the previous section) – this behavior enables a consecutive motion and when the pedestrian is in the crowd, their speed cannot reach the free flow values;

- the closer the pedestrian is to the exit, the smaller they feel (compression between pedestrians is considerable in the crowd near the exit) and the variability decreases with smaller social size (distance to the exit varies more at the beginning of the pedestrian movement, i.e. before the pedestrian join the crowd, they move freely with non-zero speed);
- statistic pedestrian speed would not be appropriate to assure that the model rules work well because it smooths the model pedestrian speed and excludes the zero values which are important for resizing of pedestrians;
- the pedestrian social size significantly decreases with increasing pedestrian individual density in time, i.e. social size responses to individual density with reverse trend, and the function $s_\alpha(t)$ has jumps right at the zero-speed points until the threshold (physical size) τ_s is reached.

We conclude this part that the pedestrian social size has the expected properties – it decreases with increasing individual density, i.e. corresponds to the conditions in the pedestrian neighborhood.

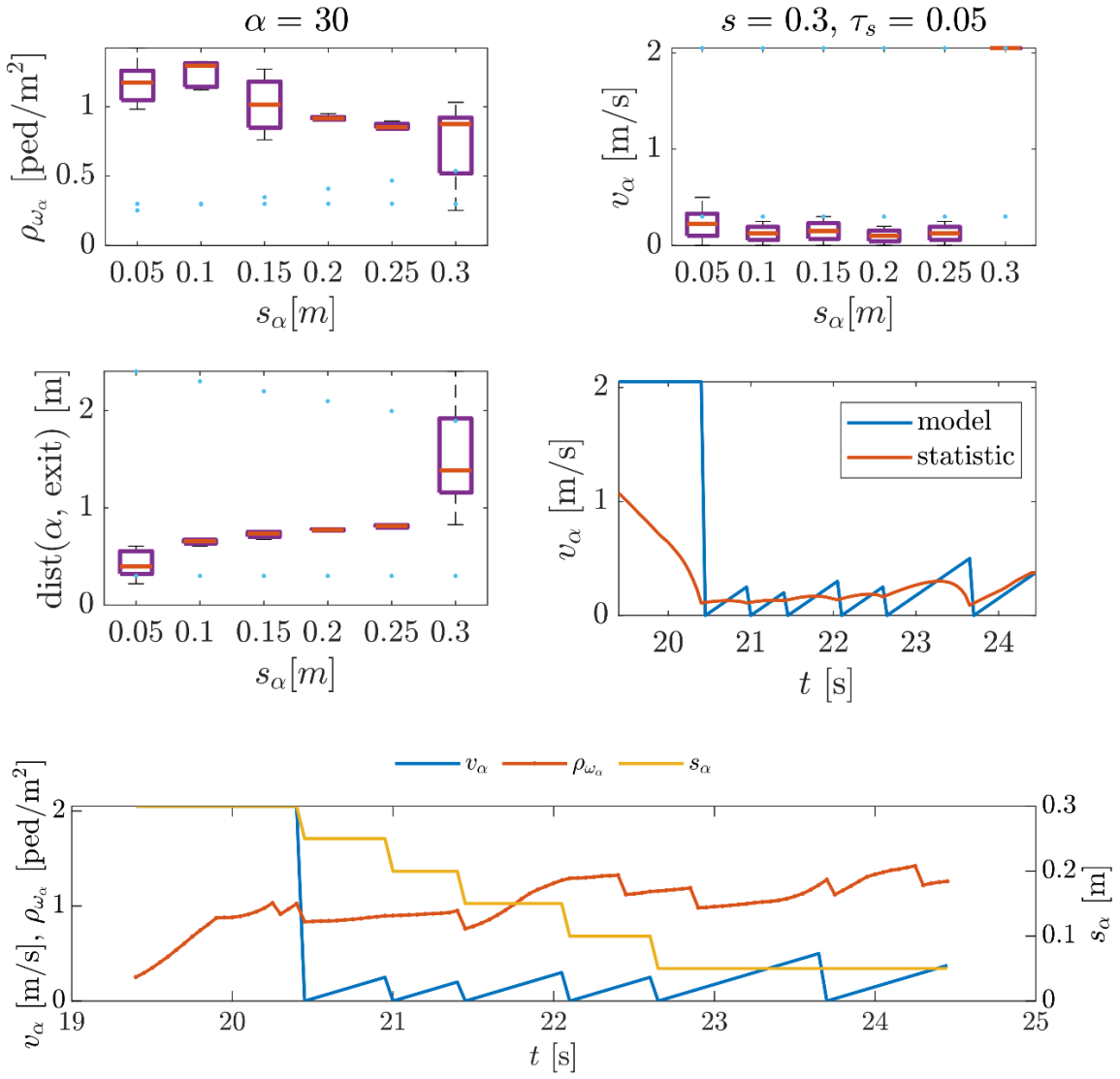

Figure 6: Individual statistics for one chosen pedestrian in the simulation $s = 0.3$ m and $\tau_s = 0.05$ m.

Spatial Localization of Pedestrian Social Size and Their Individual Density

Having studied the time development of pedestrian social size of one chosen pedestrian, we are interested in the spatial trend of social size and its response to the individual density, i.e. this is the way how to visualize data from the previous part for every pedestrian in the simulation, see Figure 7. There is depicted the simulation room including all pedestrian trajectories, colorized by values of individual density and pedestrian social size.

- The resizing of the pedestrian enables the smooth leaving process of the room – it results from the fact that the maximum individual density is around 3 ped/m^2 near the exit.
- Initial pedestrian size is rare to occur closer than 0.5 m to the exit. However, few pedestrians left the room without resizing. Change in the social size forms approximately concentric circles with center in the middle of the exit, the social size is decreasing from the outside to the inside of the circles.

We conclude this part that it is possible to leave the room without resizing if the pedestrian is fast enough (there are almost no other pedestrians in their surroundings), and the social compression is the greatest near wall.

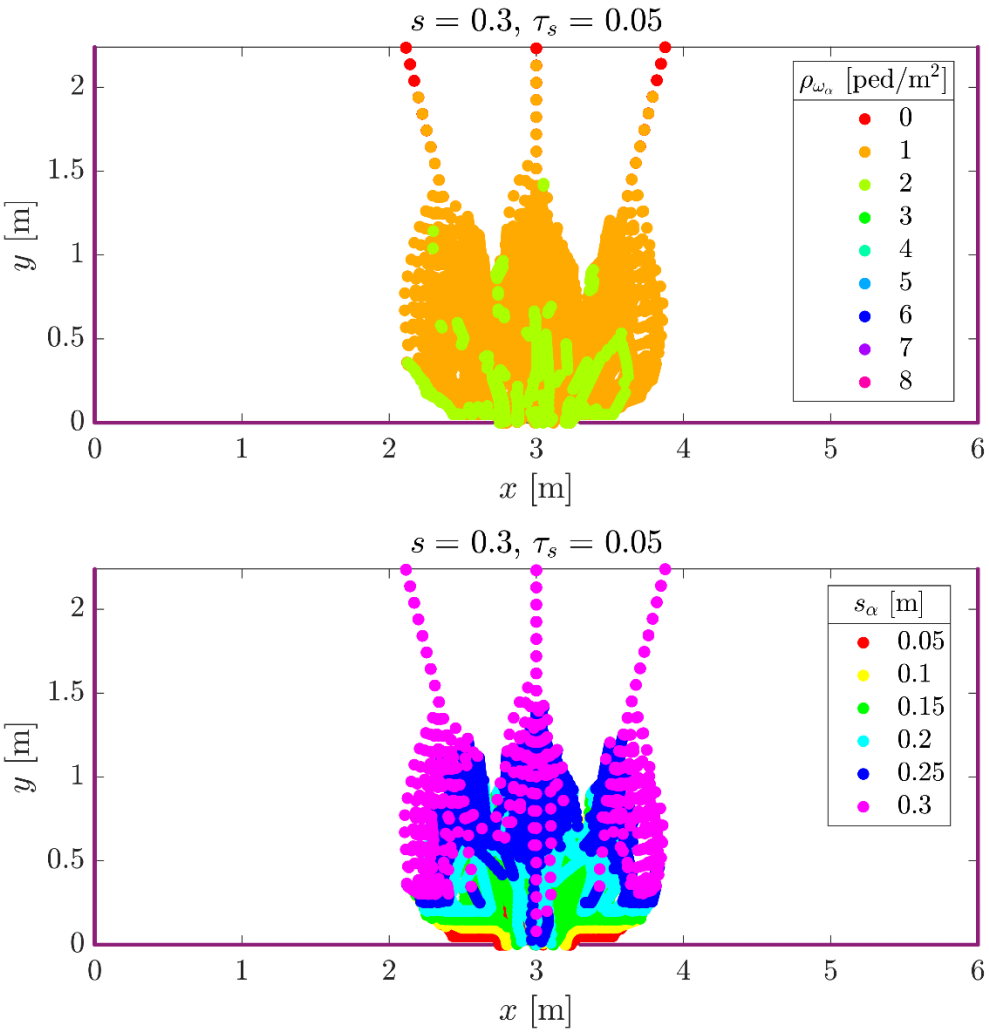

Figure 7: Spatial map of individual density and pedestrian social size near the exit in the simulation $s = 0.3\ m$ *and* $\tau_s = 0.05\ m$.

Minimum Pedestrian Size for Each Parametric Setting

So far, we have discussed only one chosen parametric setting with the largest range of possible resizing. The strength of the compression in each parametric setting is seen in Figure 8. Smaller pedestrians ($s \leq 0.18$ m) do not change their social size, i.e. minimum social size is almost equal to their initial size – it is because of the fact that there is no crowd, thus their speed is not zero and there is no reason to resize. Wider pedestrians ($s \geq 0.25$) allow more resizing than the initially smaller pedestrians - although they could leave the room much bigger, their minimum social size is around 0.09 m. It results from the fact that wider pedestrians produced congestion near the exit, so the social compression became stronger. The impact of the physical size τ_s becomes significant for $s > 0.2$ m.

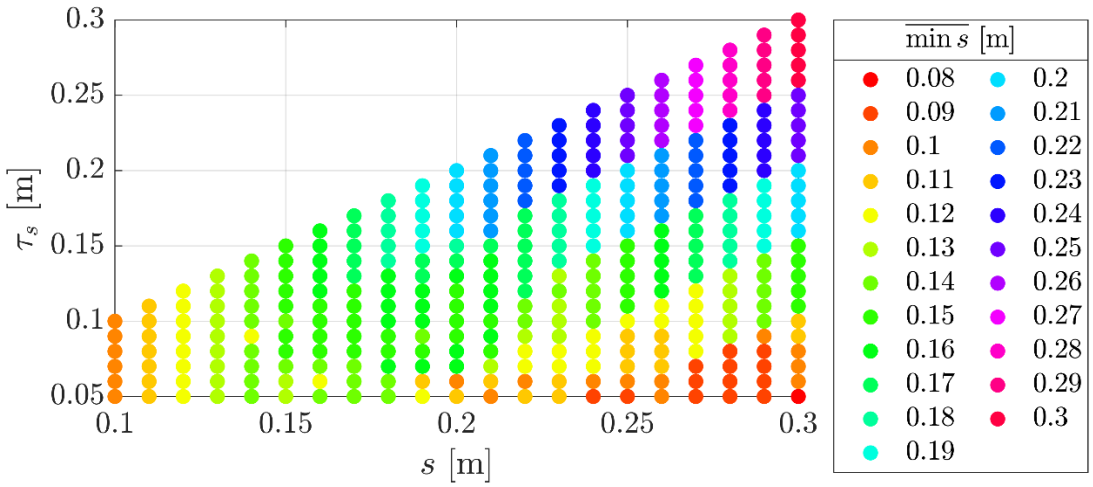

Figure 8: Mean (over pedestrians) minimum (over time) social size colorings of each input parametric combination.

MACROSCOPIC INSIGHT INTO PEDESTRIAN SIZE

At the end of the data analysis, we would like to ensure that there exists reasonable setting of two model parameters - initial and physical pedestrian size, see Figure 9. The observed quantity is mean speed (mean over both pedestrians and time).

- There is significant phase transition around $s = 0.2$ m which corresponds to the discussion mentioned above. Wider pedestrians ($s > 0.2$ m) have low values of their speed due to their initial size and the congestion that they have caused.
- Physical pedestrian size has a binary effect on the mean speed. It is possible to obtain arbitrary value of the mean speed for $\tau_s \leq 0.15$ m, the mean speed is equal to almost zero elsewhere.

We conclude this part that the prediction power of the model represented by mean speed is greatly influenced by the parametric setting of the observed parameters. For the possibility to obtain model producing data very similar to reality in the case of our inflow, it is recommended to choose values for initial and physical size of the pedestrian under restrictions $s \geq 0.16$ m (it is not only free flow) and $\tau_s \leq 0.15$ m (it is not only congested phase).

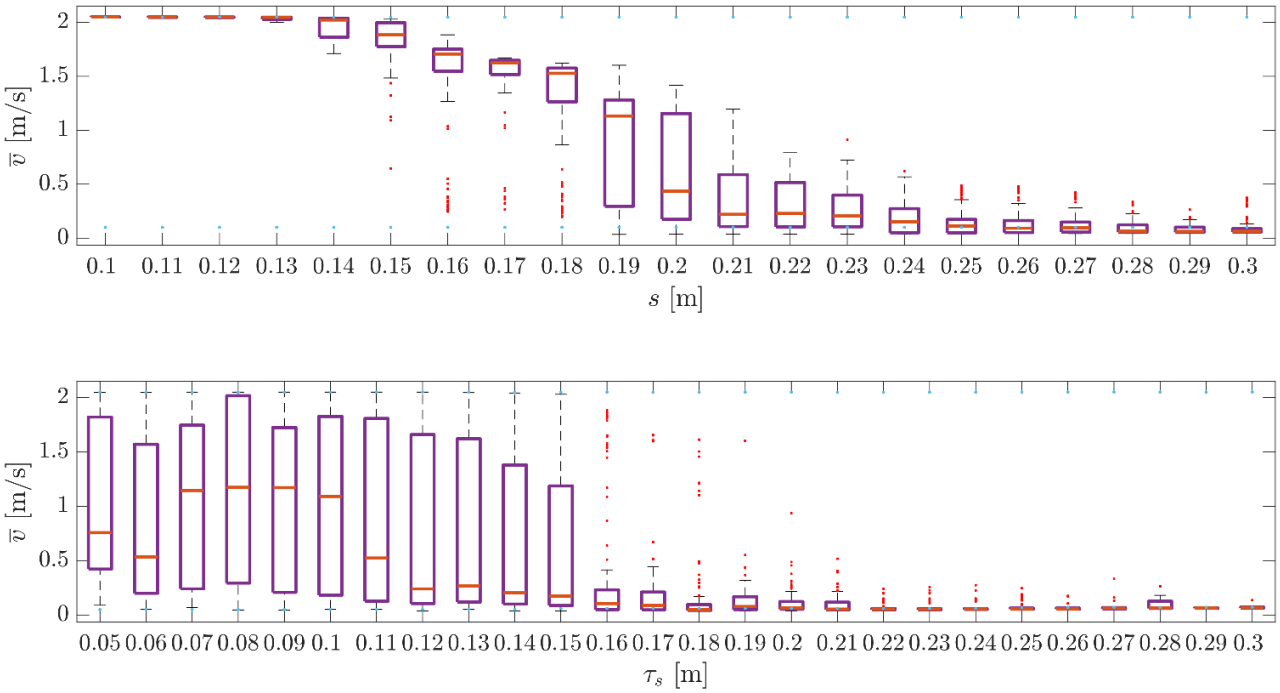

Figure 9: Mean speed over both pedestrians and time with respect to parametric settings of pedestrian (initial) size and their (physical) threshold.

CONCLUSIONS

This paper dealt with the concept of pedestrian size in pedestrian modeling, including initial, social and physical size which are mandatory for the model to run properly. These three types of pedestrian sizes are important for scaling outflow, density and speed, i.e. fundamental macroscopic quantities.

Firstly, we defined the author's decision-based model. Then we ensured that the pedestrian social size has the expected properties – it decreases with increasing individual density. Spatial change of the social size forms approximately concentric circles with the center in the middle of the exit and the social size decreases from the outside to the inside of the circles. Wider pedestrians allow more resizing than the initially smaller pedestrians - although they could leave the room bigger, they leave the room much smaller. To preserve the possibility to high model prediction power in the case of our inflow 1.5 ped/s, it is recommended to choose values for initial and physical size of the pedestrian under restrictions $s \geq 0.16$ m and $\tau_s \leq 0.15$ m.

Since the model rules are defined in the easiest way, it is expected that the concept of pedestrian sizes will produce the same results in any other model.

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